UNIT I : STRESS, STRAIN AND DEFORMATION OF SOLIDS

Rigid bodies and deformable ~~bodies~~ solids — Tension, Compression and shear Stresses — Deformation of Simple and Compound bars — Thermal Stresses — Elastic Constants — Volumetric Strains — Stresses on inclined planes — Principal stresses and Principal Planes — Mohr's Circle of Stress.

Strength of Material:

→ When an external force acts on a body, the body tends to undergo some deformation.

→ Due to Cohesion between the molecules, the body resists deformation.

↑ The action or fact of forming a united whole

→ The resistance by which the Material of the body opposes the deformation is known as strength of material.

Stress:

→ The resisting force per unit area is called stress or intensity of stress.

→ The force of resistance per unit area, offered by a body against deformation is known as stress.

→ The external force acting on the body is called the load or force.

→ The load is applied on the body while the stress is induced in the material of the body.

Mathematically, $\boxed{\sigma = \frac{P}{A}}$

Where, σ = Stress (Kgf or N ~~/m²/mm²/cm²~~)

P = External load or force

A = Cross-sectional area ($m^2/mm^2/cm^2$)

Kilo $= 10^3$

Mega $= 10^6$

Giga $= 10^9$

STRAIN:

→ When a body is subjected to some external force, there is some change of dimension of the body.

→ The ratio of change of dimension of the body to the original dimension is known as strain.

$$\text{strain} = \frac{\text{change of dimension}}{\text{original dimension}}$$

TYPES OF STRAIN:

1. Tensile strain
2. Compressive strain
3. Volumetric strain
4. Shear strain.

Tensile strain

→ The strain created due to tensile load (Pull).

Compressive strain

→ The strain created due to compressive load (Push).

Volumetric strain

→ The ratio of change of volume of the body to the original volume.

Shear strain:

→ The strain produced by shear stress is called shear strain.

TYPES OF Stresses

1. Tensile stress
2. Compressive stress
3. Shear stress.

Tensile stress

→ The stress induced in a body, when subjected to two equal and opposite pulls.

→ The tensile stress acts normal to the area and it pulls on the area.

$$\sigma = \frac{P}{A} \qquad \text{strain}\ (e) = \frac{dL}{L} = \frac{\text{change in length}}{\text{original length}}$$

Compressive Stress

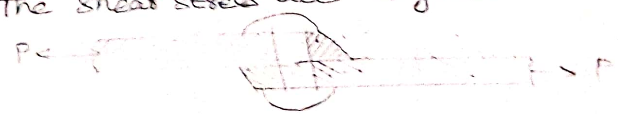
→ The stress induced in a body, when subjected to two equal and opposite pushes.

$$\text{Strain}(e) = \frac{\text{Decrease in length}}{\text{Original length}}.$$

Shear Stress

→ The stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across the section, is known as shear stress.

→ The shear stress acts tangential to the area.



ELASTICITY:

→ Any body tends to undergo some deformation when an external forces acts on it.

→ If the external force is removed and the body comes back to its original shape and size, the body is known as elastic body.

→ The property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity.

ELASTIC LIMIT:

→ The maximum limit within which, the material regain its original shape and size after the removal of load is called its elastic limit.

Hooke's law:

→ It states that "the stress is directly proportional to the strain of the material within its elastic limit".

$p \propto e$ ~~p=Ee~~ $\frac{p}{e} = E$

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YOUNG'S MODULUS (MODULUS OF ELASTICITY):

→ The ratio of stress to the corresponding strain is called Young's Modulus.

→ It is a constant and denoted by E.

$$E = \frac{\text{stress}}{\text{strain}} = \frac{\sigma}{e}$$

SHEAR MODULUS (MODULUS OF RIGIDITY).

→ The ratio of shear stress to the corresponding shear strain is called as shear modulus.

→ It is denoted by C.

$$C = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

FACTOR OF SAFETY:

It can be defined as the ratio of ultimate tensile stress to the working (or Permissible) stress.

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Permissible stress}}$$

LONGITUDINAL STRAIN:

The ratio of axial deformation to the original length of the body is called as longitudinal strain.

$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

LATERAL STRAIN:

The strain at right angles to the direction of applied load is called as lateral strain.

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$

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POISSON'S RATIO :

→ The ratio of the lateral strain to the longitudinal strain for a given material, within the elastic limit is called as poisson's ratio.

→ It is denoted by μ.

$$\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

lateral strain = μ × longitudinal strain

Since lateral strain is in a direction opposite to longitudinal strain algebrically

lateral strain = $-\mu$ × longitudinal strain

Problem 1 :-

A rod 150 cm long and of diameter 2 cm is subjected to an axial pull of 20 kN. If the modulus of elasticity of the material of the rod is 2×10^5 N/mm², determine i) stress ii) the strain iii) the elongation of the rod.

Given :

$L = 150$ cm $= 1500$ mm ; $d = 2$ cm $= 20$ mm ; $P = 20$ kN $= 20 \times 10^3$ N ; $E = 2 \times 10^5$ N/mm²

SOLUTION :

i) The stress

$$\sigma = \frac{P}{A} = \frac{20 \times 10^3}{\frac{\pi}{4}(20)^2} = 63.662 \text{ N/mm}^2.$$

ii) The strain

$$\text{strain}(e) = \frac{\sigma}{E} = \frac{63.662}{2 \times 10^5} = 0.000318.$$

iii) The elongation of the rod

$$e = \frac{dL}{L} \Rightarrow dL = e \times L = 0.000318 \times 1500$$

$$dL = \cancel{0.000318}\ 0.477 \text{ mm}$$

Problem 2 :

A bar of 30 mm diameter is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.1 mm and change in diameter is 0.004 mm. Calculate : i) Young's Modulus ii) Poisson's ratio and iii) Bulk Modulus.

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GIVEN:

$d = 30\ mm \Rightarrow A = \frac{\pi}{4}(d^2) = \frac{\pi}{4}(20)^2 =$

$P = 60\ kN = 60 \times 10^3\ N.$

Extension $(\delta L) = 0.1\ mm.$

$L = 200\ mm$

$\delta d = 0.004\ mm.$

SOLUTION:

i) Young's Modulus: (E).

$$E = \frac{\text{Stress}}{\text{Strain}}.$$

$$\text{Stress}\ (\sigma) = \frac{P}{A} = \frac{60 \times 10^3}{\frac{\pi}{4}(20)^2} = 84.87\ N/mm^2.$$

$$\text{Strain}\ (e) = \frac{\delta L}{L} = \frac{0.1}{200} = 0.0005$$

$$\therefore E = \frac{84.87}{0.0005} = 16.975 \times 10^4\ N/mm^2$$

$$\boxed{E = 1.6975 \times 10^5\ N/mm^2}$$

ii) Poisson's ratio: (μ)

$$\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}} = \frac{\left(\frac{\delta d}{d}\right)}{e} = \frac{\left(\frac{0.004}{30}\right)}{0.0005}$$

$$= \frac{0.000133}{0.0005}$$

$$\boxed{\mu = 0.266}$$

iii) Bulk modulus (K):

$$K = \frac{E}{3(1-2\mu)} = \frac{1.6975 \times 10^5}{3(1 - 2 \times 0.266)}$$

$$\boxed{K = 1.209 \times 10^5\ N/mm^2}$$

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1.6. A tensile test was conducted on a mild steel bar. The following data was obtained from the test

(i) Diameter of the steel bar = 3cm = 30 mm

(ii) Gauge length of the bar = 20cm = 200 mm

(iii) Load at elastic limit = 250 kN = 250×10^3 N

(iv) Extension at a load of 150 kN = 0.21 mm =

(v) Maximum load = 380 kN = 380×10^3 N

(vi) Total extension = 60 mm

(vii) Diameter of the rod at the failure = 2.25 cm = ~~20.25 mm~~ 22.5 mm.

Determine : (a) the Young's modulus

(b) the stress at elastic limit.

(c) the percentage elongation

(d) the % decrease in area.

Solution:

Area of rod, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(30)^2 = \underline{\underline{7.06 \times 10^2 \ mm^2}}$

(a) Youngs modulus $(E) = \frac{\sigma}{e}$

stress $\sigma = \frac{P}{A} = \frac{150 \times 10^3}{7.06 \times 10^2} = \underline{\underline{2.124 \times 10^2 \ N/mm^2}}$

strain $e = \frac{\delta L}{L} = \frac{0.21}{200} = \underline{\underline{1.05 \times 10^{-3}}}$

$E = \frac{2.124 \times 10^2}{1.05 \times 10^{-3}} = \underline{\underline{20.228 \ N/mm^2}}$

(b) Stress at elastic limit $(\sigma) = \frac{P}{A} = \frac{250 \times 10^3}{7.06 \times 10^2} = \underline{\underline{3.54 \times 10^2 \ N/mm^2}}$

(c) % elongation $= \frac{\text{Total increase in length}}{\text{original length}} \times 100$

$= \frac{60}{200} \times 100 = \underline{\underline{30\%}}$

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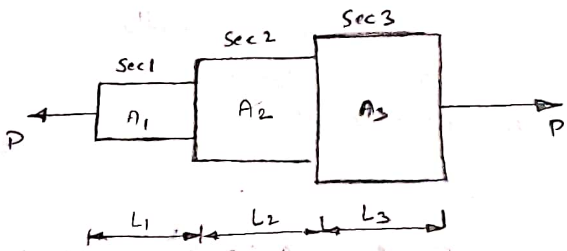
(d) % decrease in area $= \dfrac{(\text{original area} - \text{area at failure})}{\text{original area}} \times 100$

Area at failure $= \frac{\pi}{4}(D)^2 = \frac{\pi}{4}(2.25)^2 = \underline{3.97 \times 10^2\ mm^2}$

$$= \frac{(7.06 - 3.97) \times 10^2}{7.06 \times 10^2}$$

$$= \underline{\underline{43.76\ \%}}$$

Analysis of Bars of Varying Sections



P = Axial load acting on bar.

L_1 & A_1 = length & C.S. Area of section 1.

L_2 & A_2 = " " " section 2.

L_3 & A_3 = " " " section 3.

E = Youngs modulus.

Stress at section 1, 2, 3 are.

$$\sigma_1 = \frac{P}{A_1}, \quad \sigma_2 = \frac{P}{A_2}, \quad \sigma_3 = \frac{P}{A_3}.$$

Strain at section 1, 2, 3 are.

$$e_1 = \frac{\sigma_1}{E}, \quad e_2 = \frac{\sigma_2}{E}, \quad e_3 = \frac{\sigma_3}{E}$$

$$= \frac{P}{A_1 E} \qquad = \frac{P}{A_2 E} \qquad = \frac{P}{A_3 E}$$

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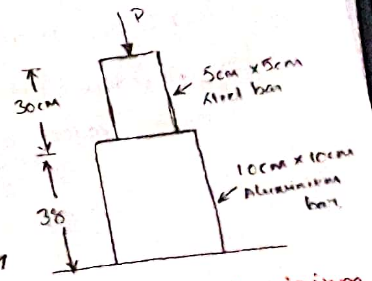
change in length of section 1, $dL_1 = e_1 L_1$

$$= \frac{PL_1}{A_1 E}$$

|||ly $dL_2 = \frac{PL_2}{A_2 E}$, $dL_3 = \frac{PL_3}{A_3 E}$

Total change in length of bar,

$$\boxed{dL = dL_1 + dL_2 + dL_3 = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]}$$



A member formed by connecting a steel bar to an aluminium bar. Assume that the bars are prevented from buckling sideways. Calculate the magnitude of force P that will cause the total length of the member to decrease 0.25 mm. The values of elastic modulus of steel is 2.1×10^5 N/mm^2 and 7×10^4 N/mm^2 for aluminium.

Given:

length of steel bar, $L_1 = 30$cm $= 300$mm

Area, $A_1 = 5 \times 5 = 25$cm $= 250$ mm^2

$E_1 = 2.1 \times 10^5$ N/mm^2

length of aluminium bar $L_2 = 38$ cm $= 380$mm

Area $A_2 = 100 \times 100 = 10000$ mm^2

$E_2 = 7 \times 10^4$ N/mm^2

Solution:

Total change in length $dL = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}\right]$

$$0.25 = P\left[\frac{300}{250 \times 2.1 \times 10^5} + \frac{380}{10000 \times 7 \times 10^4}\right]$$

$P = 224.37$ KN.

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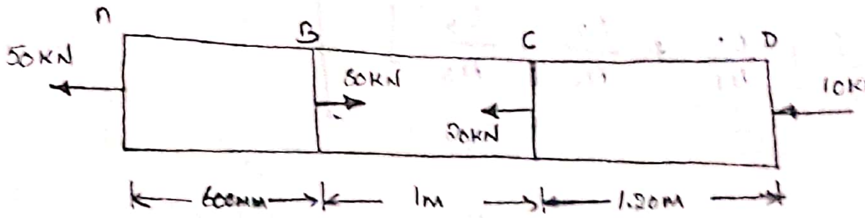
Principle of Superposition:

- when a number of loads are acting on a body, the resulting strain will be the algebraic sum of strains caused by individual loads.

Exercise problems: 1

A brass bar, having C.S. area of 1000 mm^2, is subjected to axial force.



The ultimate stress, for a hollow steel column which carries an axial load of 1.9 MN is 480 N/mm^2. If the external dia of the column is 200 mm, determine the internal dia. Take factor of safety as 4.

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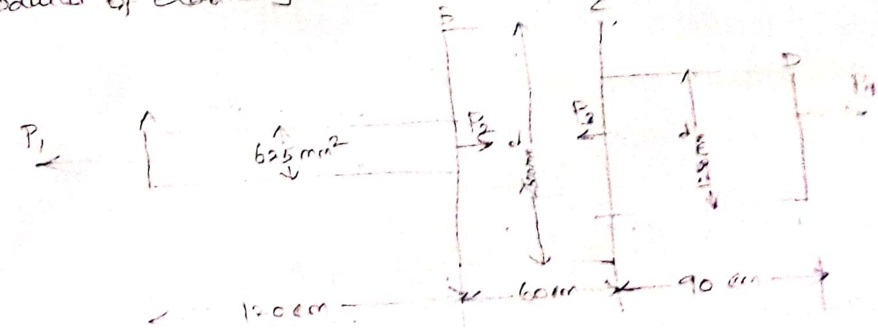
Problem 3 1.7

A member ABCD is subjected to point loads P_1, P_2, P_3 and P_4 as shown in fig. Calculate the force P_2 necessary for equilibrium if $P_1 = 45$ kN, $P_3 = 450$ kN, and $P_4 = 130$ kN. Determine the total elongation of the member, assuming the modulus of elasticity to be 2.1×10^5 N/mm².



GIVEN:

$A_1 = 625\ mm^2$ $L_1 = 120cm = 1200mm$

$A_2 = 2500\ mm^2$ $L_2 = 60cm = 600mm$

$A_3 = 1250\ mm^2$ $L_3 = 90cm = 900mm$

$E = 2.1 \times 10^5\ N/mm^2$.

SOLUTION:

Resolving the forces on the rod along its axis,

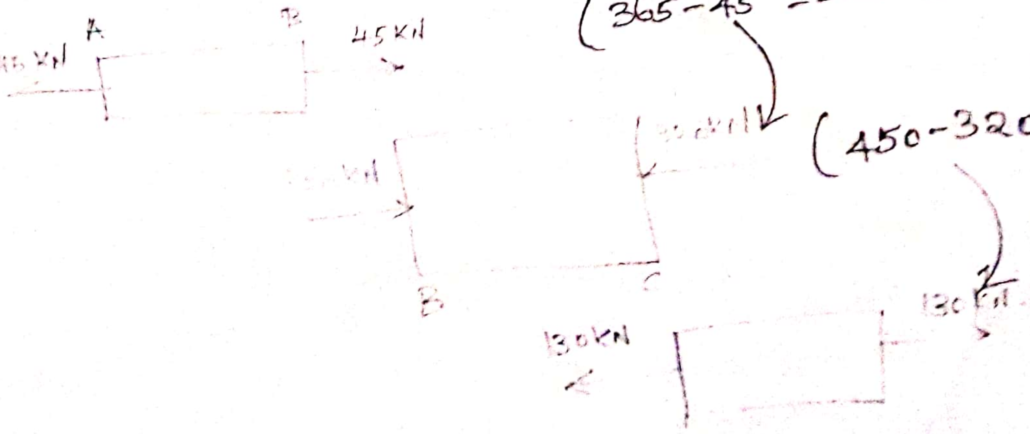
$$P_1 + P_3 = P_2 + P_4$$

$$45 + 450 = P_2 + 130$$

$$\boxed{P_2 = 365\ kN}$$

$(365 - 45 = 320kN)$.

$(450 - 320 = 130kN)$



Increase in length of AB :

$$= \frac{P}{A_1 E} \times L_1 = \frac{180000}{3125 \times 2.1\times10^5} \times 1500$$

$$= 0.4114 \text{ mm}$$

Increase in length of CD :

$$= \frac{P}{A_3 E} \times L_3 = \frac{180,000}{1250 \times 2.1\times10^5} \times 900$$

$$= 0.4457 \text{ mm}$$

Decrease in length of BC :

$$= \frac{P}{A_2 E} \times L_2 = \frac{320,000}{2500 \times 2.1\times10^5} \times 600$$

$$= 0.3657 \text{ mm}$$

$\therefore$ Total change in length of member $= 0.4114 - 0.3657 + 0.4457$

$= 0.4914$ mm (extension).

ANALYSIS OF BARS OF COMPOSITE SECTIONS :

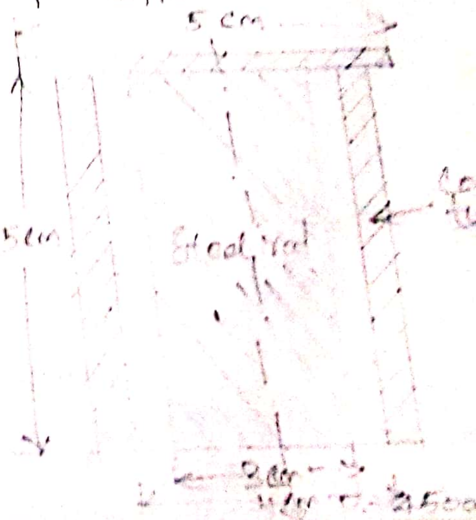
11.12.18 Problem 4x:

(1) A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4 cm. The composite bar is then subjected to an axial pull of 45000 N. If the length of each bar is equal to 15cm, determine :

i) the stresses in the rod and tube, and

ii) load carried by each bar.

Take E for steel $= 2.1\times10^5$ N/mm^2 and for copper $= 1.1\times10^5$ N/mm^2



Given :

$d = 3\text{cm} = 30\text{mm}$

$D_o = 5\text{cm} = 50\text{mm}$

$D_i = 4\text{cm} = 40\text{mm}$

$P = 45000$ N.

$L = 15\text{cm} = 150\text{mm}$.

$E_{steel} = 2.1 \times 10^5\ N/mm^2$
$E_{copper} = 1.1 \times 10^5\ N/mm^2$

SOLUTION :

Steel rod : Area $(A_s) = \frac{\pi}{4}(30)^2 = 706.86\ mm^2$

Copper tube : $(A_c) = \frac{\pi}{4}\left[50^2 - 40^2\right] = 706.86\ mm^2$

i) The stresses in the rod and tube :

Let σ_s = Stress in steel
P_s = Load carried by steel rod,
σ_c = Stress in copper,
P_c = Load carried by copper tube.

Now, Strain in steel = Strain in copper.

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c$$

$$\boxed{\sigma_s = 1.909\,\sigma_c}$$

W.K.T, Stress = $\frac{\text{load}}{\text{area}}$; Load = Stress × area.

Total load = Load on steel + load on copper.

$$= (1.909\,\sigma_c \times 706.86) + (\sigma_c \times 706.86)$$

$$45000 = (1.909\,\sigma_c \times 706.86) + (706.86)$$

$$45000 = \sigma_c\,((1.909 \times 706.86) + (706.86))$$

$$45000 = 2056.25\,\sigma_c$$

$$\boxed{\sigma_c = 21.88\ N/mm^2}$$

Sub, σ_c , We get $\sigma_s = 1.909 \times 21.88$

$$\boxed{\sigma_s = 41.77\ N/mm^2}$$

ii) Load carried by each bar :

Load = Stress × Area

$\therefore$ Load carried by steel rod, $P_s = \sigma_s \times A_s$
$= 41.77 \times 706.86$
$P_s = 29525.5\ N$

Load carried by copper tube, $P_c = 45000 - 29525.5$
$P_c = 15474.5\ N$

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Problem 5:

A compound tube consists of a steel tube 140 mm internal diameter and 160 mm external diameter and an outer brass tube 160 mm internal diameter and 180 mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900 kN. Find the stresses and the load carried by each tube and the amount it shortens. Length of each tube is 140 mm. Take E for steel as 2×10^5 N/mm^2 and for brass is 1×10^5 N/mm^2.

GIVEN:

Steel:
$d = 140$ mm
$D = 160$ mm

Brass:
$d = 160$ mm
$D = 180$ mm

$E_s = 2 \times 10^5$ N/mm^2
$E_b = 1 \times 10^5$ N/mm^2

$L_s = L_b = 140$ mm
$P = 900$ kN $= 900 \times 10^3$ N.

SOLUTION:

Area of steel (A_s) $= \frac{\pi}{4}(160^2 - 140^2) = 4712.4$ mm^2

Area of Brass $(A_b) = \frac{\pi}{4}(180^2 - 160^2) = 5340.7$ mm^2

Let, σ_s = Stress in steel in N/mm^2.
σ_b = Stress in brass in N/mm^2.

Now, strain in steel = strain in brass.

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\sigma_s = \frac{E_s}{E_b} \times \sigma_b = \frac{2 \times 10^5}{1 \times 10^5}\sigma_b$$

$$\boxed{\sigma_s = 2\sigma_b}$$

Total load = load on steel + load on brass

$$900 \times 10^3 = (\sigma_s \times A_s) + (\sigma_b \times A_b)$$

$$900 \times 10^3 = (2\sigma_b \times 4712.4) + (\sigma_b \times 5340.7)$$

$$900 \times 10^3 = 14765.5\, \sigma_b$$

$$\sigma_b = 60.95 \text{ N/mm}^2$$

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Substitute the value of σ_b, we get

$\sigma_s = 2\sigma_b$

$= 2 \times 60.95$

$\sigma_s = 121.9\ N/mm^2$

Load carried by brass tube $=$ Stress $\times$ Area

$= \sigma_b \times A_b$

$= 60.95 \times 5340.7$

$= 325515\ N = 325.515\ KN$

Load carried by Steel tube $= 900 - 325.515$

$= 574.485\ KN$

Decrease in the length of the compound tube $= \frac{\sigma_b}{E_b} \times L$

$= \frac{60.95}{1 \times 10^5} \times 140$

$= 0.0853\ mm$

(or)

Decrease in length of the compound tube $= \frac{\sigma_s}{E_s} \times L$

$= \frac{121.9}{2 \times 10^5} \times 140.$

$= 0.0853\ mm.$

13.12.15 ②

Problem 6:- Varying c.s.

The bar shown in figure is subjected to a tensile load of 160 KN. If the stress in the middle portion is limited to 150 N/mm^2, determine the diameter of the middle portion. Find also the length of the middle portion if the total elongation of the bar is to be 0.2 mm. Young's modulus is given as equal to $2.1 \times 10^5\ N/mm^2$.

Given:

$P = 160 kN = 160 \times 10^3\ N.$

$\sigma_2 = 150\ N/mm^2.$

$dL = 0.2\ mm$

$E = 2.1 \times 10^5\ N/mm^2$

$D_1 = 6\ cm = 60\ mm$

$L = 40\ cm = 400\ mm$

Area of cross-section of both end portions $(A_1) = \frac{\pi}{4} \times 60^2$

$= 2827.433 \ mm^2$

let, D_2 = Diameter of the middle portion in mm.

L_2 = length of the middle portion in mm.

$\therefore$ Length of both end portions of the bar, $L_1 = (400 - L_2)$ mm

$$\sigma = \frac{P}{A}$$

$\therefore$ for middle portion, we have $\sigma_2 = \frac{P}{A_2}$

$$150 = \frac{160 \times 10^3}{\frac{\pi}{4} D_2^2}$$

$$\therefore D_2^2 = 1358 \ mm^2$$

$$\therefore D_2 = 36.85 \ mm$$

$$D_2 = 3.685 \ cm$$

Area of cross-section of middle portion; $A_2 = \frac{\pi}{4} \times D_2^2$

$$= \frac{\pi}{4} \times (3.685)^2$$

$$= 10.66 \ cm^2$$

$$\boxed{A_2 = 1066 \ mm^2}$$

We know that,

Total change in the length of the bar, $dL = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \dots$

$$= \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2}\right]$$

$$0.2 = \frac{160000}{2.1 \times 10^5}\left[\frac{(400 - L_2)}{2827.433} + \frac{L_2}{1066}\right]$$

$$\therefore \frac{0.2 \times 2.1 \times 10^5}{160000} = \frac{(400 - L_2)}{2827.433} + \frac{L_2}{1066}$$

Simplifying, we get $L_2 = 207.14$ mm

$$\boxed{L_2 = 20.714 \ cm}$$

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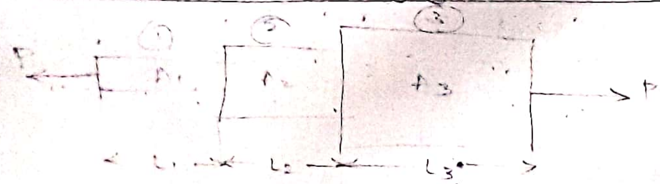
Analysis of bars of Varying cross-sections:



The stresses, strains and change in lengths will be different eventhough each section is subjected to the same axial load P.

Let,

L_1, L_2 & L_3 = lengths of section ①, ② & ③ respectively.

A_1, A_2 & A_3 = Areas of section ①, ② & ③ respectively.

E = Young's modulus for the bar.

P = Axial load acting on the bar.

Stresses: $\sigma_1 = \frac{P}{A_1}$; $\sigma_2 = \frac{P}{A_2}$; $\sigma_3 = \frac{P}{A_3}$.

Strain: $E = \frac{\sigma}{e}$ $\therefore e_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E}$

$e_2 = \frac{\sigma_2}{E} = \frac{P}{A_2 E}$

$e_3 = \frac{\sigma_3}{E} = \frac{P}{A_3 E}$.

Also, $e_1 = \frac{dL_1}{L_1}$

$\therefore dL_1 = e_1 \cdot L_1 = \frac{PL_1}{A_1 E}$ | $dL_2 = e_2 \cdot L_2 = \frac{PL_2}{A_2 E}$ | $dL_3 = e_3 \cdot L_3 = \frac{PL_3}{A_3 E}$.

$\therefore$ Total change in the length of the bar, $dL = dL_1 + dL_2 + dL_3$

$$= \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

$$dL = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right]$$

If E is different,

$$dL = P\left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3}\right]$$

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PRINCIPLE OF SUPERPOSITION :

→ According to Principle of Superposition, "When a number of loads are acting on a body, the resulting strain, will be the algebraic sum of strains caused by individual loads".

13.12.18 (11)

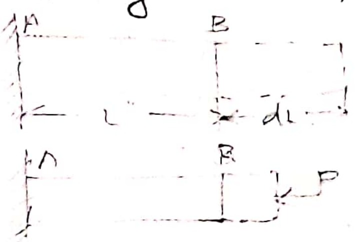
Analysis of bars of Composite sections :

→ Two or more bars of different materials and of equal lengths fixed with each other as one unit when subjected to loads is said to be a composite bar.

18.12.18

THERMAL STRESSES :

→ The stresses induced in a body due to change in temperature.



Let,

L = original length of the body,

T = Rise in temperature,

E = Young's Modulus,

α = Co-efficient of linear expansion,

dL = extension of rod due to rise of temperature.

If the rod is free to expand, the extension of the rod is given by

$$dL = \alpha \cdot T \cdot L.$$

$$\text{Compressive strain} = \frac{\text{Decrease in length}}{\text{original length}} = \frac{\alpha \cdot T \cdot L}{L + \alpha T L} \simeq \frac{\alpha T L}{L} = \alpha T$$

$$\text{Thermal strain}, e = \alpha \cdot T$$

$$\text{thermal stress } (\sigma) = \text{Thermal strain} \times E = \alpha T \times E$$

Stress and Strain When the Supports yield :

$$\text{Actual strain} = \frac{\text{Actual expansion}}{\text{Original length}} = \frac{(\alpha \cdot T \cdot L - \delta)}{L}$$

$$\text{Actual Stress} = \frac{(\alpha T L - \delta)}{L} \times E$$

Thermal stresses in composite bars:

Elastic Constants

* Poisson's ratio
* Volumetric Strains
* Bulk modulus.
* Relation b/w Young's modulus and Modulus of rigidity
* Relation b/w " " " and bulk modulus.

$\Rightarrow$ L increases (longitudinal deformation) (Axial)
other direction decreases (lateral deformation)

Longitudinal strain: $= \frac{\delta L}{L}$

Lateral strain: $= \frac{\delta b}{b}$ or $\frac{\delta d}{d}$.

Poisson's ratio:

→ The ratio of lateral strain to longitudinal strain for a given material, is called as Poisson's ratio.

→ It is denoted by μ.

$$\text{Poisson's ratio } (\mu) = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}.$$

Volumetric strain:

→ The ratio of change in volume to the original volume of a body is called Volumetric strain.

→ It is denoted by e_v.

$$e_v = \frac{\delta V}{V}$$

where, δV → change in Volume
V → original Volume.

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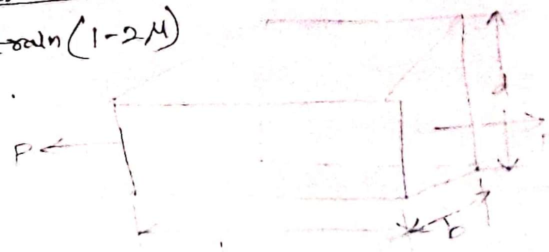
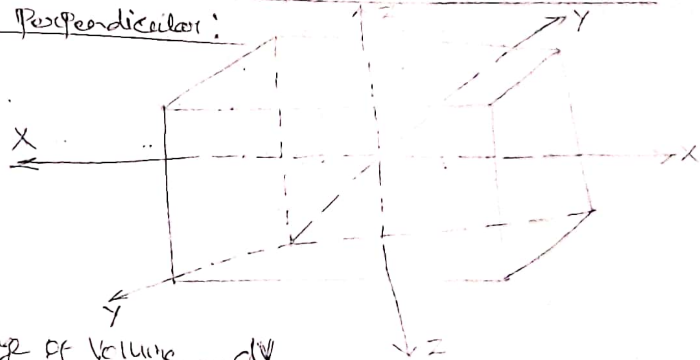
Volumetric strain of a Rectangular bar:

$$e_v = \text{longitudinal strain}(1-2\mu)$$
$$= \frac{\delta L}{L}(1-2\mu).$$



Volumetric strain of a rectangular bar subjected to three forces which are mutually perpendicular:



Volume of block, $V = xyz$.

$$\text{Volumetric strain} = \frac{\text{Change of Volume}}{\text{Original Volume}} = \frac{dV}{V}$$

If $\frac{dV}{V}$ is Positive, it represents increase in volume.

$\frac{dV}{V}$ is negative, it represents decrease in Volume.

$$\text{Volumetric strain} = \frac{dV}{V} = e_x + e_y + e_z.$$

$$\therefore \frac{dV}{V} = \frac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1-2\mu).$$

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Problem

A tensile test was conducted on a mild steel bar. The following data was obtained from the test:

i) Diameter of the steel bar = 3 cm = 30 mm
ii) Gauge length of the bar = 20 cm
iii) Load at elastic limit = 250 kN.
iv) Extension at a load of 150 kN = 0.21 mm
v) Maximum load = 380 kN.
vi) Total extension = 60 mm.
vii) Diameter of the rod at failure = 2.25 cm.

Determine: a) the Young's modulus, b) the stress at elastic limit, c) the percentage elongation, and d) the percentage decrease in area.

SOLUTION:

a) Young's modulus:

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\therefore \text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{250 \times 10^3}{\frac{\pi}{4}(30)^2} = \frac{150 \times 10^3}{706.858}$$

$$\text{Stress} = 212.206 \ N/mm^2$$

$$\text{Strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\text{Increase in length (elongation)}}{\text{original length}}$$

$$= \frac{0.21}{200} = 0.00105$$

$$\therefore E = \frac{212.206}{0.00105} = 202.1 \times 10^3 \ N/mm^2$$

b) The stress at elastic limit:

$$\text{Stress} = \frac{\text{Load at elastic limit}}{\text{Area}} = \frac{250 \times 10^3}{\frac{\pi}{4}(30)^2} = \frac{250 \times 10^3}{706.858}$$

$$\text{Stress} = 353.67 \ N/mm^2$$

c) The percentage elongation:

$$\text{Percentage elongation} = \frac{\text{Total elongation}}{\text{original length}} \times 100$$

$= \frac{60\,mm}{200\,mm} \times 100 = 30\%$

d) The Percentage decrease in area:

Percentage decrease in area $= \frac{(\text{Original area} - \text{Area at the failure})}{\text{Original area}} \times 100$.

$$= \frac{\left(\frac{\pi}{4} \times 30^2 - \frac{\pi}{4} \times 22.5^2\right)}{\frac{\pi}{4} \times 30^2} \times 100$$

$$= \frac{(706.858 - 397.607)}{706.858} \times 100$$

$$= 0.4375 \times 100$$

$$= 43.75\%$$

12.12.18
H.W

Problem: v

The ultimate stress, for a hollow steel column which carries an axial load of 1.9 MN is 480 N/mm². If the external diameter of the column is 200 mm, determine the internal diameter. Take the factor of safety as 4.

GIVEN:

Ultimate stress $= 480\ N/mm^2$

$P = 1.9\ MN = 1.9 \times 10^6\ N$

$D = 200\ mm$.

Factor of Safety $= 4$.

TO FIND:

Internal diameter (d)

SOLUTION:

Area of Cross-Section of the Column, $A = \frac{\pi}{4}(D^2 - d^2)$.

$$A = \frac{\pi}{4}(200^2 - d^2)$$

Factor of Safety $= \frac{\text{Ultimate Stress}}{\text{Working or Permissible Stress}}$.

$$4 = \frac{480}{\text{Working Stress}}$$

$\therefore$ Working Stress $(\sigma) = \frac{480}{4} = 120\ N/mm^2$.

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We know that, $\sigma = \frac{P}{A}$ or $120 = \frac{1.9 \times 10^6}{\frac{\pi}{4}(200^2 - d^2)}$

$\therefore 120 = \frac{1.9 \times 10^6 \times 4}{\pi(200^2 - d^2)}$

$40000 - d^2 = \frac{1.9 \times 10^6 \times 4}{\pi \times 120}$

$40000 - d^2 = 20159.6$

$d^2 = \cancel{60159.6} \; 19,840.4$

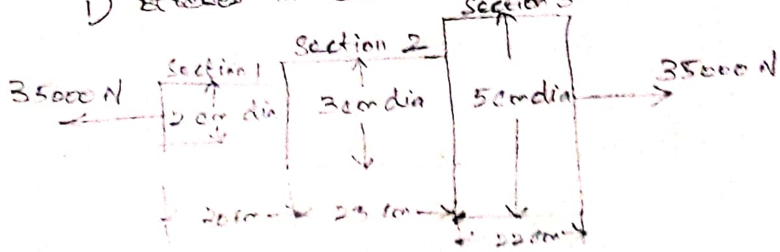
$\boxed{d = 140.85 \text{ mm}}$

ANALYSIS OF BARS OF VARYING CROSS-SECTIONS:

Problem:-

An axial pull of 35000 N is acting on a bar consisting of three lengths as shown in fig. If the Young's modulus $= 2.1 \times 10^5$ N/mm², determine:

i) Stresses in each section and ii) Total extension of the bar.



GIVEN:

Section ①:

$P = 35000$ N ; $D_1 = 2$ cm $= 20$ mm.

$A_1 = \frac{\pi}{4}(20)^2 = 314.15 \text{ mm}^2$

$L_1 = 20$ cm $= 200$ mm

Section ②:

$L_2 = 25$ cm $= 250$ mm

$D_2 = 3$ cm $= 30$ mm

$A_2 = \frac{\pi}{4}(30)^2$

$A_2 = 706.85 \text{ mm}^2$

Section ③:

$L_3 = 22$ cm $= 220$ mm

$D_3 = 5$ cm $= 50$ mm

$A_3 = \frac{\pi}{4}(50)^2$

$A_3 = 1963.495 \text{ mm}^2$

$E = 2.1 \times 10^5 \text{ N/mm}^2$

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i) Stresses in each section:

Stress in section (1), $\sigma_1 = \frac{P}{A_1} = \frac{35000}{314.15} = 111.411 \ N/mm^2$

Stress in section (2), $\sigma_2 = \frac{P}{A_2} = \frac{35000}{706.85} = 49.515 \ N/mm^2$

Stress in section (3), $\sigma_3 = \frac{P}{A_3} = \frac{35000}{1963.495} = 17.825 \ N/mm^2$

ii) Total extension of the bar:

$$\text{Total extension} = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right).$$

$$= \frac{35000}{2.1\times10^5}\left(\frac{200}{314.15} + \frac{250}{706.85} + \frac{220}{1963.495}\right)$$

$$= \frac{35000}{2.1\times10^5}\left(0.6366 + 0.3536 + 0.1120\right)$$

Total extension $= 0.1837$ mm

Principle of Superposition:

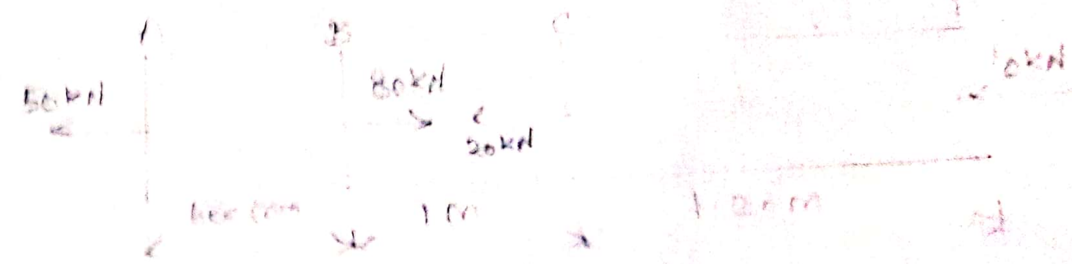
It states that, "when a number of loads are acting on a body, the resulting strain, will be the algebraic sum of strains caused by individual loads".

Total deformation of the body } = Algebraic sum of deformations of the individual sections.

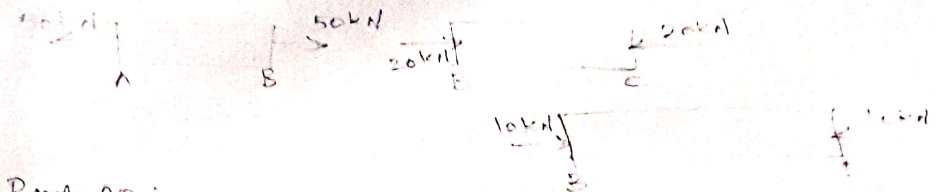
13.12.16 Problem:

(1) A brass bar, having cross-sectional area of 1000 mm², is subjected to axial forces as shown in fig. Find the total elongation of the bar. Take $E = 1.05\times10^5 \ N/mm^2$.

GIVEN:



$A = 1000\ mm^2$.
$E = 1.05 \times 10^5\ N/mm^2$.
Let, dL = Total elongation of the bar.



Part AB:

Tensile load : $50\ kN = 50 \times 10^3\ N$

Increase in length of AB $= \frac{P_1}{AE} \times L_1$

$= \frac{50 \times 10^3}{1000 \times 1.05 \times 10^5} \times 600$

$= 0.2857\ mm$.

Part BC:

Compressive load : $20\ kN = 20 \times 10^3\ N$.

Decrease in length of BC $= \frac{P_2}{AE} \times L_2 = \frac{20{,}000}{1000 \times 1.05 \times 10^5} \times 1000$

$= 0.1904\ mm$.

Part BD:

Compressive load : $10\ kN = 10 \times 10^3\ N$.

Decrease in length of BD $= \frac{P_3}{AE} \times L_3 = \frac{10 \times 10^3}{1000 \times 1.05 \times 10^5} \times 2200$

$= 0.2095$

Total elongation of bar $= 0.2857 - 0.1904 - 0.2095$ (Increase → +ve, Decrease → −ve)

$= -0.1142\ mm$.

14.12.18 (5)

ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD:

Total extension, $dL = \frac{4PL}{\pi E D_1 D_2}$

Total extension, $dL = \frac{4PL}{\pi E D^2}$ (uniform dia rod)

where, P = Axial tensile load on the bar.
L = Total length of the bar
E = Young's Modulus.

18.12.18 PROBLEM:

Find the modulus of elasticity for a rod, which tapers uniformly from 30mm to 15mm diameter in a length of 350 mm. The rod is subjected to an axial load of 5.5 KN and extension of the rod is 0.025 mm.

GIVEN:

$D_1 = 30mm$

$D_2 = 15\ mm$

$L = 350\ mm$.

$P = 5.5\ KN = 5.5\times10^3\ N$.

$dL = 0.025\ mm$

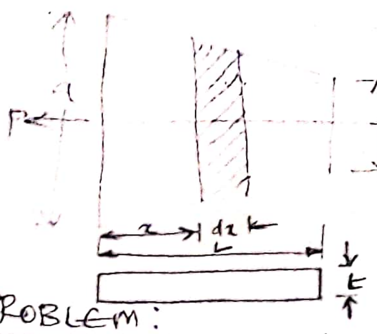
SOLUTION: $dL = \dfrac{4PL}{\pi E D_1 D_2} = \dfrac{4\times5.5\times10^3\times350}{\pi\times E\times30\times15}$

$$\therefore E = \frac{4\times5.5\times10^3\times350}{\pi\times0.025\times30\times15}$$

$$= 217975.937\ N/mm^2$$

$$E = 2.17975\times10^5\ N/mm^2$$

18.12.18 ANALYSIS OF UNIFORMLY TAPERING RECTANGULAR BAR:



a = Width at bigger end

b = Width at smaller end

L = Length of the bar

t = thickness of the bar

$$\therefore \text{Total extension} = \frac{PL}{Et(a-b)}\log_e\frac{a}{b}.$$

18.12.18 PROBLEM:

A rectangular bar made of steel is 2.8m long and 15mm thick. The rod is subjected to an axial tensile load of 40KN. The width of the rod varies from 75 mm at one end to 30mm at the other. Find the extension of the rod if $E = 2\times10^5\ N/mm^2$.

GIVEN:

Length, $L = 2.8m = 2800\ mm$ | $a = 75mm$

$t = 15\ mm$ | $b = 30mm$

$P = 40\times10^3\ N$. | $E = 2\times10^5\ N/mm^2$.

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TO FIND:

Extension of the rod (dL).

SOLUTION:

$$dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b}$$

$$= \frac{40\times10^3 \times 2800}{2\times10^5 \times 15 (75-30)} \log_e\left(\frac{75}{30}\right)$$

$$= 0.8296 \times 0.9163$$

$$dL = 0.76 \text{ mm}$$

Analysis of bars of Composite Sections:

14/8/18

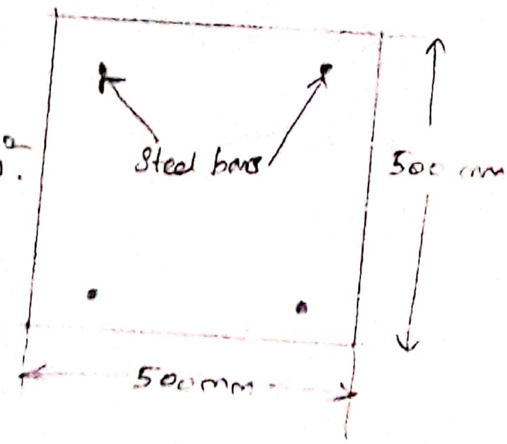
(1) Problem:

A load of 2 MN is applied on a short concrete column 500 mm × 500 mm. The column is reinforced with four steel bars of 10 mm diameter, one in each corner. Find the stresses in the concrete and steel bars. Take E for steel as 2.1×10^5 N/mm^2 and for concrete as 1.4×10^4 N/mm^2.

GIVEN:

$P = 2$ MN $= 2\times10^6$ N

Area $= 500 \times 500 = 250000$ mm^2.



SOLUTION:

Area of 4 steel bars, A_s

$$A_s = 4 \times \frac{\pi}{4}(10)^2$$

$$A_s = 314.159 \text{ mm}^2$$

Area of Concrete (A_c) = Area of column − Area of steel bars

$= 250000 - 314.159$

$= 249685.841$ mm^2

Let, σ_s = Stress in steel bar in N/mm^2

σ_c = Stress in Concrete in N/mm^2

Now, Strain in Steel = Strain in Concrete

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(2)

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{2.1\times10^5}{1.4\times10^4}\sigma_c$$

$$\boxed{\sigma_s = 15\sigma_c} \quad \text{------(i)}$$

Now, Total load = Load on steel + load on concrete

$$P = \sigma_s \cdot A_s + \sigma_c A_c$$

$$2000000 = (15\sigma_c \times 314.159) + (\sigma_c \times 249685.841)$$

$$2000000 = 254398\,\sigma_c$$

$$\boxed{\therefore \sigma_c = 7.86\ N/mm^2}$$

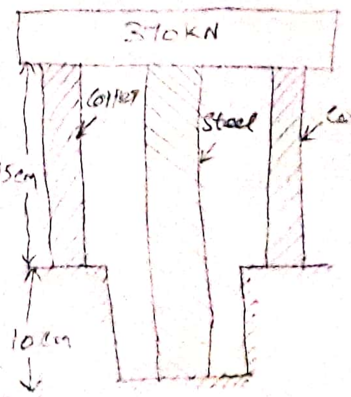
Substitute σ_c value in (i), we get

$$\sigma_s = 15 \times 7.86$$

$$\boxed{\sigma_s = 117.92\ N/mm^2}$$

Problem:

14.12.18 (b) A steel rod and two copper rods together support a load of 370KN as shown in fig. The cross-sectional area of steel rod is 2500 mm² and of each copper rod is 1600 mm². Find the stresses in the rods. Take E for steel $= 2\times10^5$ N/mm² and for copper $= 1\times10^5$ N/mm²



Given:

Load, $P = 370KN = 370\times10^3 N$

$A_s = 2500 mm^2$

$A_c = 1600 mm^2$

$\therefore A_c = 2\times1600 = 3200\ mm^2$

$E_s = 2\times10^5\ N/mm^2$

$E_c = 1\times10^5\ N/mm^2$

Length of steel rod, $L_s = 15+10$
$= 25\ cm = 250\ mm.$

Length of copper rods, $L_c = 15\ cm = 150\ mm.$

Solution: Let σ_s = stress in steel rod in N/mm²
σ_c = stress in copper rods in N/mm²

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We know that,

Decrease in the length of the steel rod = Decrease in the length of copper rods.

Decrease in length of steel rod = Strain in steel rod × length of steel rod

$$= \frac{\text{Stress in Steel}}{E_s} \times L_s$$

$$= \frac{\sigma_s}{2\times10^5} \times 250 \quad \text{———(i)}$$

Decrease in length of copper rods = Strain in Copper rods × length of Copper rods

$$= \frac{\text{Stress in Copper}}{E_c} \times L_c$$

$$= \frac{\sigma_c}{1\times10^5} \times 150 \quad \text{———(ii)}$$

Equating the above two, we get

$$\frac{\sigma_s}{2\times10^5} \times 250 = \frac{\sigma_c}{1\times10^5} \times 150$$

$$\sigma_s = \sigma_c \times \frac{2\times10^5}{1\times10^5} \times \frac{150}{250}$$

$$\boxed{\sigma_s = 1.2\,\sigma_c} \quad \text{———(iii)}$$

Also, Total load applied = load on Steel + load on Copper

$$3{,}10{,}000 = \sigma_s A_s + \sigma_c A_c$$

$$3{,}10{,}000 = (1.2\,\sigma_c \times 2500) + (\sigma_c \times 3200)$$

$$310000 = 6200\,\sigma_c$$

$$\sigma_c = \frac{310000}{6200}$$

$$\boxed{\sigma_c = 59.67\ N/mm^2}$$

Substitute σ_c value in (iii), we get

$$\sigma_s = 1.2 \times \sigma_c = 1.2 \times 59.67$$

$$\boxed{\sigma_s = 71.604\ N/mm^2}$$

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Thermal Stresses in Composite bars:

Load on the brass = Stress × Area

$= \sigma \times A$

Actual expansion (1) = free expansion + Expansion due to tensile stress

$= \alpha T L + \frac{\sigma}{E} \cdot L$

Actual expansion (2) = free expansion − Contraction due to Compressive stress

$= \sigma T L - \frac{\sigma}{E} \cdot L$

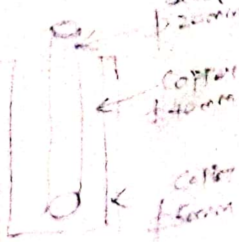
Actual expansion (1) = Actual expansion (2)

$$\alpha T L + \frac{\sigma}{E} \times L = \alpha T L - \frac{\sigma}{E} \times L$$

$$\alpha T + \frac{\sigma}{E} = \alpha T - \frac{\sigma}{E}$$

Problem:

A steel rod of 20mm diameter passes centrally through a Copper tube of 50mm external diameter and 40mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of the rod. If the temperature of the assembly is raised by 50°c, Calculate the stresses developed in Copper and Steel. Take E for Steel and Copper as 200 GN/m² and 100 GN/m² and α for steel and copper as 12×10^{-6} per°c and 18×10^{-6} per°c.



Given:

Dia = 20mm.

Area of steel rod, $A_s = \frac{\pi}{4} \times 20^2 = 100\pi \text{ mm}^2$

Area of copper tube, $A_c = \frac{\pi}{4}(50^2 - 40^2) = 225\pi \text{ mm}^2$

Rise of temperature, $T = 50°c$.

$E_s = 200 \times 10^9 \text{ N/m}^2 = 200 \times 10^3 \text{ N/mm}^2$

$E_c = 100 \times 10^9 \text{ N/m}^2 = 100 \times 10^3 \text{ N/mm}^2$

$\alpha_s = 12 \times 10^{-6}$ per°c

$\alpha_c = 18 \times 10^{-6}$ per°c

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Free expansion of copper tube will be more than the common expansion.

Free expansion of steel rod will be less than the common expansion.

Let σ_s = Tensile stress in steel.
σ_c = Compressive stress in copper.

for the equilibrium of the system,
Tensile load on steel = Compressive load on copper.

$$\sigma_s \cdot A_s = \sigma_c \cdot A_c$$

$$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$= \frac{225\pi}{100\pi} \times \sigma_c$$

$$= 2.25\,\sigma_c$$

Actual expansion of steel = Actual expansion of copper.

$\therefore$ Actual expansion of steel = free expansion of steel + Expansion due to tensile stress in steel.

$$= \alpha_s T L + \frac{\sigma_s}{E_s} L$$

Actual expansion of copper = free expansion of copper - Contraction due to compressive stress in copper.

$$= \alpha_c T L - \frac{\sigma_c}{E_c} L$$

$$\therefore \alpha_s T L + \frac{\sigma_s}{E_s} L = \alpha_c T L - \frac{\sigma_c}{E_c} L$$

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

$$(12 \times 10^{-6} \times 50) + \left(\frac{2.25\,\sigma_c}{200 \times 10^3}\right) = (18 \times 10^{-6} \times 50) - \left(\frac{\sigma_c}{100 \times 10^3}\right)$$

$$\therefore \frac{2.25\sigma_c}{200 \times 10^3} + \frac{\sigma_c}{100 \times 10^3} = (18 \times 10^{-6} \times 50) - (12 \times 10^{-6} \times 50)$$

(ii)

$$(1.125\times10^{-5}\,\sigma_c)+(10^{-5}\,\sigma_c)=6\times10^{-6}\times 50$$

$$2.125\times10^{-5}\,\sigma_c=30\times10^{-5}$$

$$2.125\,\sigma_c=30$$

$$\sigma_c=\frac{30}{2.125}=14.117\ N/mm^2$$

$$\boxed{\sigma_c=14.117\ N/mm^2}$$

Substitute σ_c in $\sigma_s=2.25\,\sigma_c$, we get

$$\sigma_s=2.25\times14.117$$

$$\boxed{\sigma_s=31.76\ N/mm^2}$$

ELASTIC CONSTANTS:

Here, L [illegible]

* Both axial and lateral deformation occurs in the above.
* This chapter deals with the above deformations. They are,

i) Poisson's ratio
ii) Volumetric strains.
iii). Bulk modulus.
iv) Relation between Young's Modulus and modulus of rigidity. (Rigidity modulus).
v) Relation between Young's Modulus and bulk modulus.

Poisson's ratio varies from 0.25 to 0.33

Poisson's ratio:

→ The ratio of the lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and is denoted by μ.

lateral strain = $-\mu\times$ longitudinal strain

$$\mu=\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

The value of Poisson's ratio varies from 0.25 to 0.33.
The value ranges from 0.45 to 0.50 for rubber.

Volumetric Strain:

→ The ratio of change in volume to the original volume of a body is called volumetric strain.
→ It is denoted by e_v.

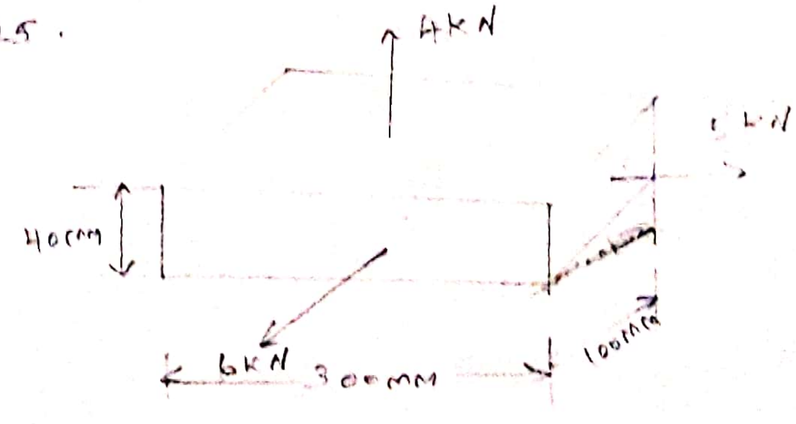
Mathematically, $e_v = \frac{\delta V}{V}$.

Where, δV → change in volume.
V → original volume.

Volumetric strain of a rectangular bar subjected to three forces which are mutually perpendicular.

Problem:

A metallic bar 300 mm X 100 mm X 40 mm is subjected to a force of 5 KN (tensile), 6 KN (tensile) and 4 KN (tensile) along x, y and z directions respectively. Determine the change in the volume of the block. Take $E = 2 \times 10^5 \ N/mm^2$ and Poisson's ratio = 0.25.



Given:

Dimensions of the bar = 300 mm X 100 mm X 40 mm = x X y X z

$\therefore V = x \times y \times z$

$= 300 \times 100 \times 40$

Volume, $V = 1200000 \ mm^3$

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Load in x - direction $= 5 kN = 5 \times 10^3$ N.
Load in y - direction $= 6 kN = 6 \times 10^3$ N.
Load in Z - direction $= 4 kN = 4 \times 10^3$ N.
$E = 2 \times 10^5$ N/mm^2.
Poisson's ratio, $\mu = 0.25$.

Solution:

∴ Stress in x-direction, $\sigma_x = \dfrac{\text{Load in x-direction}}{y \times z}$

$= \dfrac{5 \times 10^3}{100 \times 40}$

$\boxed{\sigma_x = \cancel{5 \times 10^2}\ 1.25 \text{ N/mm}^2}$

Stress in y-direction, $\sigma_y = \dfrac{\text{Load in y-direction}}{x \times z}$

$= \dfrac{6 \times 10^3}{300 \times 40}$

$\boxed{\sigma_y = 0.5 \text{ N/mm}^2}$

Stress in Z-direction, $\sigma_z = \dfrac{\text{Load in z-direction}}{x \times y}$

$= \dfrac{4 \times 10^3}{300 \times 100}$

$\boxed{\sigma_z = 0.133 \text{ N/mm}^2}$

Volumetric strain is given by,

$\dfrac{dV}{V} = \dfrac{1}{E}(\sigma_x + \sigma_y + \sigma_z)(1 - 2\mu)$.

$= \dfrac{1}{2 \times 10^5}(1.25 + 0.5 + 0.133)(1 - 2 \times 0.25)$

$\dfrac{dV}{V} = \dfrac{1.883}{2 \times 10^5 \times 2}$

∴ $dV = \dfrac{1.883}{4 \times 10^5} \times V$

$= \dfrac{1.883}{4 \times 10^5} \times 1200000$

$\boxed{dV = 5.649 \text{ mm}^3}$

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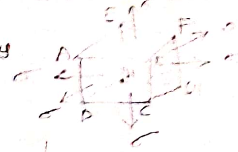
Bulk Modulus:

→ The ratio of direct stress to the corresponding volumetric strain is found to be constant for a given material when the deformation is within a certain limit.



→ This ratio is known as Bulk modulus and is usually denoted by K.

→ Mathematically, it is denoted by

$$\boxed{K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\left(\frac{dV}{V}\right)}}.$$

Relation between Young's Modulus (E) and Bulk Modulus (K):

Bulk Modulus, $K = \frac{E}{3(1-2\mu)}$.

Young's modulus, $E = 3K(1-2\mu)$.

From the above equation, we obtain Poisson's ratio as,

$$1-2\mu = \frac{E}{3K}$$

$$3K(1-2\mu) = E$$

$$1-2\mu = \frac{E}{3K}$$

$$\mu = \frac{\left(1-\frac{E}{3K}\right)}{2}$$

$$\boxed{\mu = \frac{3K-E}{6K}}$$

Relationship between Young's Modulus (E) and Modulus of rigidity (C):

We know that, $C = \frac{\text{Shear stress}}{\text{Shear strain}}$.

After deriving we get, $C = \frac{E}{2(1+\mu)}$.

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Principal Stresses and Principal planes :

→ In many engineering problems both direct (tensile or compressive stress) and shear stresses are acting at the same time.

→ In such situation, the resultant stress across any section will be neither normal nor tangential to the plane.

→ Here, the stresses, acting on an inclined plane (or oblique section). will be analysed.

Principal planes :

→ The planes, which have no shear stress, are known as principal planes.

→ Hence principal planes are the planes of zero shear stress.

→ These planes carry only normal stresses.

Principal Stresses :

The normal stresses, acting on a principal plane, are known as principal stresses.

Methods for determining stresses on oblique section :

1. Analytical method   2. Graphical method.

Analytical method :

1. A member subjected to a direct ~~tensile~~ stress in one plane.
2. The member is subjected to like direct stresses in two mutually perpendicular directions.

A member subjected to a direct stress in one plane:

(σ) σ_n, σ_t

Normal stress, $\sigma_n = \sigma \cos^2\theta$

Tangential stress, $\sigma_t = \frac{\sigma}{2} \sin 2\theta$.
(shear stress)

Maximum shear stress $= \frac{\sigma}{2}$

1) A rectangular bar of cross-sectional area 10000 mm² is subjected to an axial load of 20KN. Determine the normal and shear stresses on a section which is inclined at an angle of 30° with normal cross-section of the bar.

Given:

$A = 10000\ mm^2$; $P = 20\times10^3\ N$; $\theta = 30°$

Solution:

Direct stress $(\sigma) = \frac{P}{A} = \frac{20\times10^3}{10000} = 2\ N/mm^2$.

We know that, normal stress $(\sigma_n) = \sigma\cos^2\theta$

$= 2\times\cos^2 30$

$= 2\times 0.866^2$ $(\because \cos 30° = 0.866)$

$\sigma_n = 1.5\ N/mm^2$

Shear stress $(\sigma_t) = \frac{\sigma}{2}\sin 2\theta$

$= \frac{2}{2}\times\sin 2(30)$

$= \sin 60$

$= 0.866\ N/mm^2$.

2) Find the diameter of a circular bar which is subjected to an axial pull of 160KN, if the maximum allowable shear stress on any section is 65 N/mm².

Given:

$P = 160\times10^3\ N$

$\sigma_{max} = 65\ N/mm^2$.

To Find:

D

Solution:

Area of the bar $(A) = \frac{\pi}{4}(D^2)$

Direct stress$(\sigma) = \frac{P}{A} = \frac{160\times10^3}{\frac{\pi}{4}D^2} = \frac{640000}{\pi D^2}\ N/mm^2$.

Maximum shear stress $= \frac{\sigma}{2} = \frac{640000}{2\times\pi D^2}$.

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$\therefore 65 = \dfrac{64000}{2 \times \frac{\pi}{4} D^2}$ ($\because$ maximum shear stress $= 65\ N/mm^2$)

$D^2 = \dfrac{64000}{2 \times \frac{\pi}{4} \times 65}$

$D^2 = 1567$

$D = 39.58$ mm

A member subjected to like direct stresses in two mutually perpendicular directions:

Normal stress $(\sigma_n) = \dfrac{\sigma_1 + \sigma_2}{2} + \dfrac{\sigma_1 - \sigma_2}{2} \cos 2\theta$

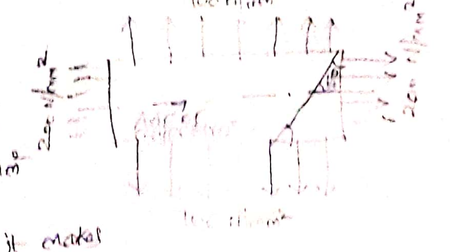
Tangential (shear) stress $(\sigma_t) = \dfrac{(\sigma_1 - \sigma_2)}{2} \sin 2\theta$

Maximum shear stress $(\sigma_t)_{max} = \dfrac{\sigma_1 - \sigma_2}{2}$ ($\because \theta = 45°$)

Resultant stress $(\sigma_R) = \sqrt{\sigma_n^2 + \sigma_t^2}$

Angle made by resultant stress with normal of oblique plane $(\tan\phi) = \dfrac{\sigma_t}{\sigma_n}$

3) The stresses at a point in a bar are 200 N/mm² (tensile) and 100 N/mm² (compressive). Determine the resultant stress in magnitude and direction on a plane inclined at 60° to the axis of the major stress. Also determine the maximum intensity of shear stress in the material at that point.



Given:

Major principal stress, $\sigma_1 = 200\ N/mm^2$

Minor principal stress, $\sigma_2 = -100\ N/mm^2$

Angle of the plane, which it makes with the major principal stress $= 60°$

Solution:

Angle $(\theta) = 90 - 60 = 30°$

Resultant Stress in magnitude and direction:

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$\therefore$ Normal stress $(\sigma_n) = \dfrac{\sigma_1+\sigma_2}{2} + \dfrac{\sigma_1-\sigma_2}{2}\cos 2\theta$

$$= \frac{200+(-100)}{2} + \frac{200-(-100)}{2}\cos(2\times 30^\circ).$$

$$= \frac{200-100}{2} + \frac{200+100}{2}\cos 60^\circ.$$

$$= 50 + 150\cos 60^\circ$$

$$= 50 + 150(0.5)$$

$$= 50 + 75$$

$$\boxed{\sigma_n = 125\ N/mm^2}$$

Tangential stress $(\sigma_t) = \dfrac{(\sigma_1-\sigma_2)}{2}\sin 2\theta = \dfrac{200-(-100)}{2}\sin(2\times 30^\circ)$

$$= \frac{200+100}{2}\times \sin 60^\circ$$

$$= 150 \times 0.866$$

$$\boxed{\sigma_t = 129.9\ N/mm^2}$$

$\therefore$ Resultant stress $(\sigma_R) = \sqrt{125^2 + 129.9^2}$

$$\boxed{\sigma_R = 180.27\ N/mm^2}$$

The inclination of the resultant stress with the normal of the inclined plane is given by,

$$\tan\phi = \frac{\sigma_t}{\sigma_n} = \frac{129.9}{125} = 1.04$$

$$\therefore \phi = \tan^{-1}(1.04)$$

$$\boxed{\phi = 46^\circ 6'}$$

Maximum shear stress:

The equation is given by,

$$(\sigma_t)_{max} = \frac{\sigma_1-\sigma_2}{2} = \frac{200-(-100)}{2}$$

$$= \frac{300}{2}$$

$$\boxed{(\sigma_t)_{max} = 150\ N/mm^2}$$

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1) At a point in a strained material the principal stresses are 100 N/mm² (tensile) and 60 N/mm² (compressive). Determine the normal stress, shear stress and resultant stress on a plane inclined at 50° to the axis of major principal stress. Also determine the maximum shear stress at the point

Given:

Major principal stress, $\sigma_1 = 100\ N/mm^2$

Minor principal stress, $\sigma_2 = -60\ N/mm^2$

Angle of the inclined plane with the axis of major principal stress: 50°

Solution:

Angle of the inclined plane with the axis of minor principal stress = 40°

$$\theta = 90 - 50$$

$$\theta = 40^\circ$$

Normal stress (σ_n):

$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$= \frac{100 + (-60)}{2} + \frac{100 - (-60)}{2} \cos 2(40^\circ)$$

$$= 20 + 80 \cos 80^\circ$$

$$\boxed{\sigma_n = 33.89\ N/mm^2}$$

Shear stress (σ_t):

$$\sigma_t = \frac{\sigma_1 - \sigma_2}{2} \sin 2\theta = \frac{100 - (-60)}{2} \sin 2(40^\circ)$$

$$= 80 \times \sin 80$$

$$\boxed{\sigma_t = 78.785\ N/mm^2}$$

Resultant stress (σ_R):

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2} = \sqrt{33.89^2 + 78.785^2}$$

$$= \sqrt{1148.53 + 6207.07}$$

$$\boxed{\sigma_R = 85.765\ N/mm^2}$$

Maximum shear stress:

$$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100 - (-60)}{2}$$

$$\boxed{(\sigma_t)_{max} = 80\ N/mm^2}$$

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A member subjected to a simple shear stress

Normal stress $(\sigma_n) = \tau \cdot \sin 2\theta$

Tangential stress $(\sigma_t) = -\tau \cdot \cos 2\theta$

A member subjected to direct stresses in two mutually perpendicular directions accompanied by a simple shear stress

Normal stress $(\sigma_n) = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta + \tau \cdot \sin 2\theta$

Tangential stress $(\sigma_t) = \frac{\sigma_1 - \sigma_2}{2} \cdot \sin 2\theta - \tau \cos 2\theta$

Position of principal planes:

$\sigma_t = 0$

$\tan 2\theta = \frac{2\tau}{(\sigma_1 - \sigma_2)}$

Major principal stress $= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

Minor principal stress $= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

Maximum shear stress $(\sigma_t)_{max} = \pm \frac{1}{2} \sqrt{(\sigma_2 - \sigma_1)^2 + 4\tau^2}$

$= \frac{1}{2} \sqrt{(\sigma_2 - \sigma_1)^2 + 4\tau^2}$

$(\sigma_t)_{max} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$

5) Direct stresses of 120 N/mm² tensile and 90 N/mm² compression exist on two perpendicular planes at a certain point in a body. They are also accompanied by shear stress on the planes. The greatest principal stress at the point due to these is 150 N/mm².

a) What must be the magnitude of the shearing stresses on the two planes?

b) What will be the maximum shearing stress at the point?

Given:

Major tensile stress, $\sigma_1 = 120\ N/mm^2$,

Minor compressive stress, $\sigma_2 = -90\ N/mm^2$.

Greatest principal stress $= 150\ N/mm^2$

SOLUTION:

~~Maximum~~ Major Principal Stress $= \dfrac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\dfrac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$$150 = \frac{120 + (-90)}{2} + \sqrt{\left(\frac{120 - (-90)}{2}\right)^2 + \tau^2}$$

$$150 = \frac{120 - 90}{2} + \sqrt{\left(\frac{210}{2}\right)^2 + \tau^2}$$

$$\therefore\ 150 - 15 = \sqrt{(105)^2 + \tau^2}$$

$$135 = \sqrt{(105)^2 + \tau^2}$$

Squaring both sides, we get

$$135^2 = 105^2 + \tau^2$$

$$\tau^2 = 135^2 - 105^2$$

$$= 18225 - 11025$$

$$\tau^2 = 7200$$

$$\therefore \tau = \sqrt{7200} \Rightarrow \boxed{\tau = 84.853\ N/mm^2}$$

Maximum shear stress at that point:

Maximum shear stress $(\sigma_{t\,max}) = \dfrac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$

$$= \frac{1}{2}\sqrt{(120 - (-90))^2 + 4 \times 7200}$$

$$= \frac{1}{2}\sqrt{210^2 + 28800}.$$

$$= \frac{1}{2} \times 270$$

$$\boxed{(\sigma_t)_{max} = 135\ N/mm^2}$$

RESULT:

Magnitude of shear stress $(\tau) = 84.853\ N/mm^2$,

Maximum shear stress at the point $(\sigma_t)_{max} = 135\ N/mm^2$.

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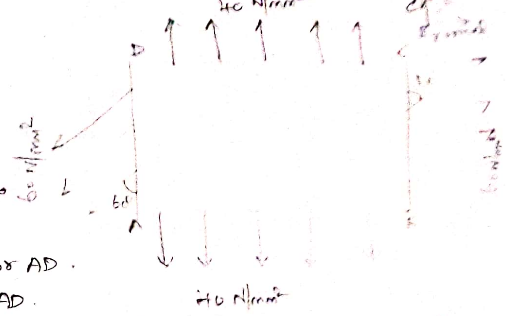
6) A Point in a Strained material is subjected to the stress as shown in fig. Locate the Principal planes, and evaluate the Principal Stresses.



Given:

The stress on the face BC or AD is normal.

Stress can be resolved into two components.

i) normal to the face BC or AD.

ii) Along the face BC or AD.

$\therefore$ Stress normal to the face BC or AD $= 60 \times \sin 60° = 51.96\ N/mm^2$

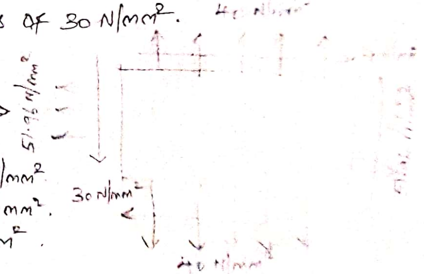
Stress along the face BC or AD $= 60 \times \cos 60° = 30\ N/mm^2$.

The stress along the face BC or AD is called shear stress.

$\therefore \tau = 30\ N/mm^2$.

Due to complementary shear stress the face AB and CD will also be subjected to shear stress of $30\ N/mm^2$.

Stresses acting on the material $\Rightarrow$



Major tensile stress, $\sigma_1 = 51.96\ N/mm^2$.

Minor tensile stress, $\sigma_2 = 40\ N/mm^2$.

Shear stress, $\tau = 30\ N/mm^2$.

Location of Principal Planes:

Let θ = angle, which one of the Principal Planes make with the stress of $40\ N/mm^2$.

The location of Principal Planes is given by the equation,

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} = \frac{2 \times 30}{51.96 - 40} = 4.999$$

$$\therefore 2\theta = \tan^{-1}(4.999)$$

$$2\theta = 78° 42' \text{ or } 258° 42'$$

$$\therefore \theta = 39° 21' \text{ or } 129° 21'$$

Principal stress:

$\therefore$ Major Principal Stress $= \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$.

$= \frac{51.96+40}{2} + \sqrt{\left(\frac{51.96-40}{2}\right)^2 + 80^2}$.

$= 45.98 + 30.6$

$= 76.58 \ N/mm^2$.

$\therefore$ Minor Principal Stress $= \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$= \frac{51.96+40}{2} - \sqrt{\left(\frac{51.96-40}{2}\right)^2 + 30^2}$

$= 45.98 - 30.6$

$= 15.38 \ N/mm^2$

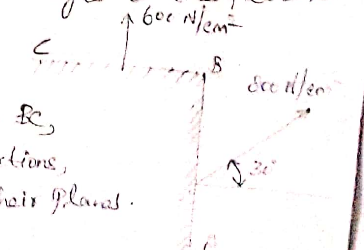
7) The intensity of resultant stress on a plane AB at a point in a material under stress is $800 \ N/cm^2$ and it is inclined at $30°$ to the normal to that plane. The normal component of stress on another plane BC at right angles to the plane AB is $600 \ N/cm^2$.

Determine the following:

i) the resultant stress on the plane BC,

ii) the principal stresses and their directions,

iii) the maximum shear stresses and their planes.



GIVEN:

Resultant Stress on Plane AB $= 800 \ N/cm^2$.

Angle of inclination $(\theta) = 30°$

Normal Stress on Plane BC $= 600 \ N/cm^2$

SOLUTION:

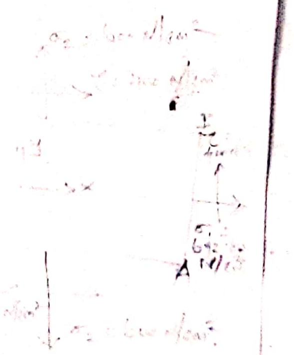
Normal Stress on Plane AB $= 800 \times \cos 30°$

$= 692.82 \ N/cm^2$

$\sigma_1 = 692.82 \ N/cm^2$

Tangential Stress on Plane AB $= 800 \times \sin 30°$

$= 400 \ N/cm^2$.



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Shear stress $(\tau_{AB}) = 400\ N/cm^2$.

$\tau_{BC} = 400\ N/cm^2$.

i) Resultant Stress on plane BC :

$\sigma_2 = 600\ N/cm^2$.

$\tau = 400\ N/cm^2$.

$\therefore$ Resultant Stress on plane BC $= \sqrt{\sigma_2^2 + \tau^2}$

$= \sqrt{600^2 + 400^2}$

$= 721\ N/cm^2$.

The resultant will be inclined at an angle θ with the horizontal given by,

$\tan\theta = \frac{\sigma_2}{\tau} = \frac{600}{400} = 1.5$

$\theta = \tan^{-1}(1.5) = 56.3°$

ii) Principal Stresses and their directions :

$\therefore$ Major Principal Stress $= \frac{\sigma_1 + \sigma_2}{2} + \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$= \frac{692.82 + 600}{2} + \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$

$= 1049.09\ N/cm^2$ (Tensile).

$\therefore$ Minor Principal Stress $= \frac{\sigma_1 + \sigma_2}{2} - \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$

$= \frac{692.82 + 600}{2} - \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$

$= 243.73\ N/cm^2$

Direction of Principal Stresses: $\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2}$

$= \frac{2 \times 400}{(692.82 - 600)}$

$= \frac{800}{92.82}$

$\tan 2\theta = 8.618$

$\therefore 2\theta = \tan^{-1}(8.618)$

$= 83.38°$ or

$2\theta = 263.38°$

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$\therefore \theta = 41.69°$ (or) $131.99°$

iii) The maximum shear stresses and their planes :

$$\therefore (\sigma_t)_{max} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}$$

$$= \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2}$$

$$= \sqrt{\left(\frac{692.82 - 600}{2}\right)^2 + 400^2}$$

$$(\sigma_t)_{max} = 402.68 \ N/cm^2.$$

Mohr's circle :

→ It is a graphical method for finding normal, tangential and resultant stresses on an oblique plane.

→ It can be drawn for three cases namely,

A body subjected to two mutually perpendicular

i) Principal tensile stresses of unequal intensities.

ii) Principal stresses which are unequal and unlike (i.e. one tensile and other compressive)

iii) Principal tensile stresses accompanied by a simple shear stress.

CASE I :

Mohr's circle :

1) Take any point 'A' & draw a horizontal line through 'A'. Take $AB = \sigma_1$ & $AC = \sigma_2$

2) With BC as diameter draw a circle.

3) Let O is the centre of the circle.

4) Through O, draw a line OE making an angle 2θ with OB.

5) From E, draw ED ⊥r on AB.

6) Join AE. Now, normal & tangential stresses are given by AD and ED respectively.

7) Resultant stress is given by AE.

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Radius of Mohr's circle $= \frac{\sigma_1 - \sigma_2}{2}$

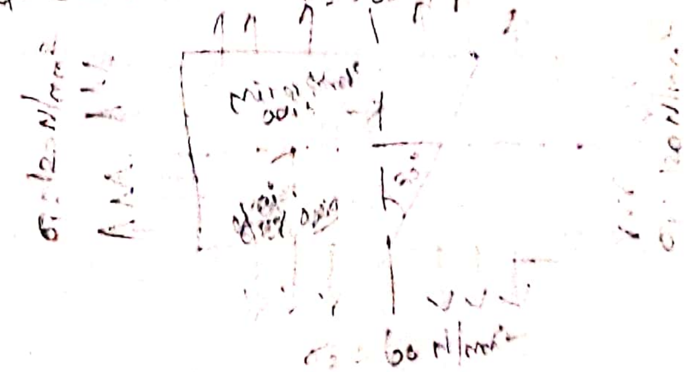
Maximum shear stress = radius of the Mohr's circle.

Normal stress $(\sigma_n) = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2}\cos 2\theta$.

Tangential stress $(\sigma_t) = \frac{\sigma_1 - \sigma_2}{2}\sin 2\theta$.

$(\sigma_t)_{max} = \frac{\sigma_1 - \sigma_2}{2}$.

1. The tensile ~~of compressive~~ stresses at a point across two mutually perpendicular planes are 120 N/mm² and 60 N/mm². Determine the normal, tangential and resultant stresses on a plane inclined at 30° to the axis of the minor stress by Mohr's circle method.



Given:

$\sigma_1 = 120\ N/mm^2$ (Tensile)

$\sigma_2 = 60\ N/mm^2$ (tensile).

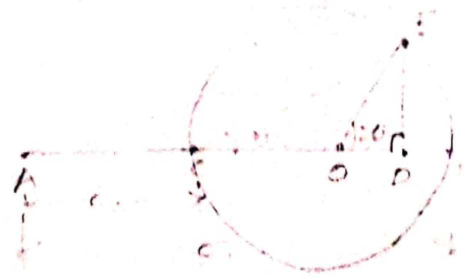
$\theta = 30°$

Solution:

Scale: let 1cm = 10 N/mm².

$\sigma_1 = \frac{120}{10} = 12$ cm.

$\sigma_2 = \frac{60}{10} = 6$ cm.



i) Take any point 'A' & draw a horizontal line through 'A'.

ii) Take 'AB' $= \sigma_1 = 12$ cm and $AC = \sigma_2 = 6$ cm.

iii) With BC as diameter describe a circle ($\because BC = \sigma_1 - \sigma_2 = 12 - 6 = 6$ cm).

iv) Let O is the centre of the circle. Through O draw a line OE making an angle 2θ. ($\because 2\theta = 2 \times 30 = 60°$)

v) From E draw ED ⊥r to CB. Join AE. Measure lengths AD, ED, and AE.

By measurements,

length AD = 10.5 cm

" ED = 2.6 cm

" AE = 10.82 cm

Normal stress = length AD × Scale = 10.5 × 10 = 105 N/mm².

Tangential Stress = length ED × Scale = 2.6 × 10 = 26 N/mm².

Resultant Stress = length AE × Scale = 10.82 × 10 = 108.2 N/mm².

CASE II :

A body subjected to two mutually ⊥r Principal Stresses which are unequal and unlike (i.e one is tensile and other is Compressive).

Let ;

σ_1 = major Principal tensile Stress,

σ_2 = minor Principal Compressive Stress,

θ = Angle made by the oblique Plane with the axis of minor Principal Stress.

Mohr's circle :

i) Take any Point A and draw a horizontal line through A on both sides of A.

ii) Take AB = σ_1 (+) towards right of A and AC = σ_2 (-) towards left side of A to some suitable scale.

iii) Bisect BC at O.

iv) with O as centre and radius equal to CO or OB draw a circle.

v) Through O draw a line OE making an angle 2θ with OB.

vi) From E, draw ED perpendicular to AB. Join AE and CE. Then normal and shear stress on the oblique plane are given by AD and ED. Length AE represents the resultant stress on the oblique plane.

length AD = Normal stress on oblique plane

ED = Shear stress on oblique plane,

AE = Resultant stress on oblique plane

Angle φ = obliquity

Radius of Mohr's circle $= \frac{\sigma_1 + \sigma_2}{2}$

Normal stress $(\sigma_n) = \frac{\sigma_1 - \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2} \cos 2\theta$.

Tangential stress $(\sigma_t) = \frac{\sigma_1 + \sigma_2}{2} \sin 2\theta$.

<u>Case III :</u>

Mohr's circle when a body is subjected to two mutually perpendicular principal tensile stresses accompanied by a simple shear stress:

i) Take any point A and draw a horizontal line through A.

ii) Take $AB = \sigma_1$ and $AC = \sigma_2$ towards right of A to same suitable scale.

iii) Draw perpendiculars at B and C and cutoff BF and CG equal to shear stress τ to the same scale.

iv) Bisect BC at O. Now with O as centre and radius equal to OG or OF draw a circle.

v) Through O, draw a line OE making an angle 2θ with OF as shown.

vi) From E, draw ED $\perp$ to CB. Join AE.

vii) Now AE = Resultant stress.

AD = Normal stress

ED = Shear stress.

<u>Problem:</u>

At a certain point in a strained material, the intensities of stresses on two planes at right angles to each other are $20\,N/mm^2$ and $10\,N/mm^2$ both tensile. They are accompanied by a shear stress of magnitude $10\,N/mm^2$. Find graphically, the location of principal planes and evaluate the principal stresses.

<u>Given:</u>

$\sigma_1 = 20\,N/mm^2$

$\sigma_2 = 10\,N/mm^2$

$\tau = 10\,N/mm^2$.

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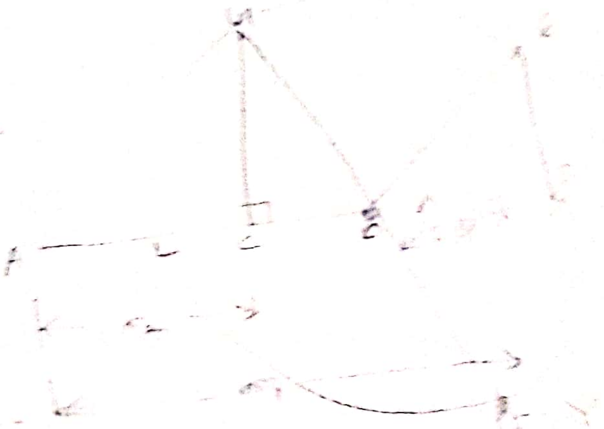
Solution:

Take Scale 1 cm = 2 N/mm²

$\sigma_1 = \frac{20}{2} = 10$ cm

$\sigma_2 = \frac{10}{2} = 5$ cm

$\tau = \frac{10}{2} = 5$ cm.



AM = Major Principal Stress = [illegible]

AL = Minor Principal Stress = [illegible]

[illegible] = 6[illegible]

Major Principal Stress = length [illegible]

= [illegible] × 2

= 26 [illegible] N/mm²

Minor Principal Stress = length AL × [illegible]

= [illegible] × 2

= [illegible] N/mm²

Location of Principal Planes:

$2\theta = 6$[illegible] $\Rightarrow \theta =$ [illegible]

The second principal plane is [illegible]

$\theta + 90° =$ [illegible]

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Problem 1:

A rod of 2m long at a temp of 10°C. Find the expansion of the rod, when the temperature is raised to 80°C. If this expansion is prevented, find the stress induced in the material of rod. Take $E = 1.0 \times 10^6$ MN/m^2 and $\alpha = 0.000012$ per degree centigrade.

Given data:

Length of the rod, $L = 2m = 2000$ mm.

Initial Temperature, $t_1 = \cancel{100}\ 10°C$

Final Temperature, $t_2 = 80°C$

$\therefore$ Rise in temperature, $T = (t_2 - t_1) = 70°C$

Youngs modulus, $E = 1 \times 10^6$ MN/m^2

$= 1 \times 10^6$ N/mm^2

Co-efficient of linear expansion, $\alpha = 0.000012$

To find:

(i) Expansion of the rod, $dL = ?$

(ii) Stress induced in the rod, $\sigma = ?$

Solution:

(i) Expansion, $dL = \alpha . T . L$

$= 0.000012 \times 70 \times 2000$

$\boxed{dL = 1.68 \text{ mm}}$

(ii) Thermal stress, $\sigma = \alpha . T . E$

$= 0.000012 \times 70 \times 1 \times 10^6$

$\boxed{\sigma = 840 \text{ N/mm}^2}$

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Problem 2:

A steel rod of 3 cm dia and 5 m long is connected to two grips and the rod is maintained at a temperature of 95°C. Determine the stress and pull exerted when the temperature falls to 30°C, if

(i) the ends do not yield, and

(ii) the ends yield by 0.12 cm.

Take $E = 1 \times 10^6$ MN/m² and $\alpha = 0.000012 /°C$

Given data:

Dia. of steel rod, $D = 3cm = 30mm$

Length of the rod, $L = 5m = 5000 mm$.

Initial temperature, $t_1 = 95°C$

Final temperature $t_2 = 30°C$

∴ Fall in temperature, $T = t_1 - t_2 = 95 - 30$

$= 65°C$

Youngs modulus $= 1 \times 10^6 \, MN/m^2 = 1 \times 10^6 \, N/mm^2$

Co-efficient of Linear expansion $\alpha = 0.000012 /°C$

Solution:

(i) When the ends do not yield

Area $= \frac{\pi}{4} D^2$

$= \pi/4 \times 30^2$

$\boxed{A = 706.85 \, mm^2}$

Stress, $\sigma = \alpha . T . E$

$= 0.000012 \times 65 \times 1 \times 10^6$.

$\cancel{\sigma = 1.720 \, N/mm^2}$ $\boxed{\sigma = 780 \, N/mm^2}$

Pull in rod, P = stress x Area $= 780 \times 706.85$

$\boxed{P = 5.51 \times 10^5 \, N}$

(ii) when the ends yield by 0.12 cm = 1.2 mm.

Stress, $\sigma = \frac{\alpha . T . L - \delta}{L} \times E = \frac{(0.000012 \times 65 \times 5000 - 1.2)}{5000} \times 1 \times 1$

$\boxed{\sigma = 540 \, N/mm^2}$

Pull in the rod = stress x Area $= 540 \times 706.85$

$\boxed{P = 3.8 \times 10^5 \, N}$

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$= \frac{P}{A.E} = \frac{30000}{600 \times 2\times10^5}$

$= 0.00025$

longitudinal strain $= \frac{\delta L}{L}$

$\delta L = 0.00025 \times 4000$

$\boxed{\delta L = 1.0 \text{ mm}}$

Poissons ratio $= \frac{\text{Lateral strain}}{\text{longitudinal strain.}}$

$0.3 = \frac{\text{Lateral strain}}{0.00025}$

$\therefore$ Lateral strain $= 0.3 \times 0.00025$

$\boxed{= 0.000075}$

lateral strain $= \frac{\delta b}{b}$ (or) $\frac{\delta d}{d}$ $\left(\text{or } \frac{\delta t}{t}\right)$

$\delta b = b \times$ lateral strain

$= 30 \times 0.000075$

$\boxed{\delta b = 0.00225 \text{ mm.}}$

$\delta t = t \times$ lateral strain.

$= 20 \times 0.000075$

$\boxed{\delta t = 0.0015 \text{ mm.}}$

Volumetric strain of a Rectangular Bar which is subjected to an axial load P in the direction of its length.

Let, $\delta L =$

$\delta b =$

$\delta d =$

$\therefore$ Final length of the bar $= L + \delta L$

" width " $= b + \delta b$

" depth " $= d + \delta d$

Original volume of bar $V = L.d.b$

Final volume $= (L+\delta L)(b+\delta b)(d+\delta d)$

$= L.b.d + bd\,\delta L + Lb\,\delta d + Ld.\delta b$

change in volume δV = Final volume − Original volume.

$=$

Volumetric strain $e_V = \dfrac{\delta V}{V}$

$$= \frac{bd.\delta L + Ld.\delta b + Lb.\delta d}{L.b.d}$$

$$= \frac{\delta L}{L} + \frac{\delta b}{b} + \frac{\delta d}{d}$$

e_V = longitudinal strain + 2 (lateral strain)

lateral strain $= -\mu \times$ longitudinal strain.

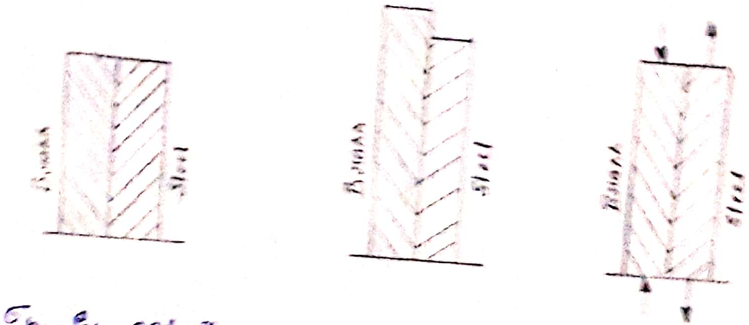
e_V = longitudinal strain $(1-2\mu)$

$$= \frac{\delta L}{L}(1-2\mu).$$

$\times 1\times 10^{6}$

$P = 0.0\ldots$

Thermal Stress in composite Bar



A_b, σ_b, e_b and α_b = Area, Stress, Strain and Co-efficient of linear expansion of ~~steel~~ brass

A_s, σ_s, e_s and α_s = Area, Stress, strain and Co-efficient of linear expansion of Steel.

E_b = Young's modulus of brass

E_s = Young's modulus of steel.

δ = Actual expansion of Composite bar.

For equilibrium of system, compression in brass should be equal to tension in steel.

$\therefore$ Load on brass = Load on steel

$$P_b = P_s$$

$$\sigma_b \times A_b = \sigma_s \times A_s$$

Also, Actual expansion of steel = Actual expansion of brass.

Free expansion of steel + Expansion due to tensile stress in steel = Free expansion of Copper − Contraction due to compressive stress induced in brass

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$$\alpha_s \cdot T \cdot L + \frac{\sigma_s}{E_s} \cdot L = \alpha_b \cdot T \cdot L - \frac{\sigma_b}{E_b} \cdot L$$

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_b T - \frac{\sigma_b}{E_b}$$

Problem 3:

A steel rod of 20mm dia passes centrally through a copper tube of 50 mm external dia. and 40 mm internal dia. The tube is closed at each end by rigid plates of negligible thickness. (The nuts are tightened lightly home on the projecting parts of the rod.) If the temperature of the assembly is raised by 50°C, Calculate the stresses developed in copper and steel. Take E for steel and copper as 200 GN/m² and 100 GN/m² and α for steel and copper as 12×10^{-6} per °C and 18×10^{-6}/°C.

Given:

Dia. of the steel rod, D_s = 20mm

Ext dia of Copper tube D_{e1} = 50 mm

Internal dia. of Copper tube D_{e2} = 40 mm.

Rise in temp, T = 50°C

Youngs modulus of steel, $E_s = 200\ GN/m^2 = 200\ kN/mm^2$

Youngs modulus of Copper, $E_c = 100\ GN/m^2 = 100\ kN/mm^2$

Co-efficient for the linear expansion
for steel $\alpha_s = 12 \times 10^{-6}/°C$
for copper $\alpha_c = 18 \times 10^{-6}/°C$

To find:

Stress developed in steel, σ_s = ?

" " " Copper, σ_c = ?

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H.W

Problem 4:

A Steel tube of 30 mm external dia and 20 mm internal dia encloses a copper rod of 15 mm dia to which it is rigidly joined at each end. If, at a temperature of 10°C there is no longitudinal stresses, calculate the stresses in the rod and tube when temp. is raised to 200°C. Take E for steel & copper as 2.1×10^5 N/mm² and 1×10^5 N/mm² respectively. The value of co-efficient of linear expansion for steel & copper is 11×10^{-6} /°C & 18×10^{-6} /°C.

Problems on Elastic Constants

(1) Determine the change in length, breadth and thickness of a steel bar which is 4m long, 30mm wide and 20 mm thick and is subjected to an axial pull of 30 KN in the direction of its length. Take $E = 2 \times 10^5$ N/mm² & Poissons ratio = 0.3.

Given data:

Length of the bar, $L = 4m = 4000$ mm

Breadth of the bar, $b = 30$ mm

Thickness " , $t = 20$ mm

$\therefore$ Area of C.S, $A = b \times t$

$= 30 \times 20 = 600 \text{ mm}^2$

Axial Pull, $P = 30$ KN

Youngs modulus, $E = 2 \times 10^5$ N/mm²

Poisson's ratio $\mu = 0.3$

Solution

Strain in the direction of the load (longitudinal strain),

$$= \frac{\text{Stress}}{E} = \frac{\text{load}}{\text{Area} \times E}$$

Solution

Tensile load on steel = Tensile load on copper.

$$\sigma_s . A_s = \sigma_c . A_c$$

$$\sigma_s = \frac{A_c}{A_s} . \sigma_c$$

$$= \frac{\pi/4 (50^2 - 40^2)}{\pi/4 (20)^2} . \sigma_c$$

$$\sigma_s = \frac{706.85}{314.1} \sigma_c$$

$$\boxed{\sigma_s = 2.25 \sigma_c} \quad \text{——— (1)}$$

Actual Expansion in steel = Actual expansion in copper

$$\alpha_s . T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

$$12\times10^{-6} \times 50 + \frac{2.25 \sigma_c}{200\times10^3} = 18\times10^{-6} \times 50 - \frac{\sigma_c}{100\times10^3}$$

$$6\times10^{-4} + \frac{2.25 \sigma_c}{200\times10^3} = 9\times10^{-4} - \frac{\sigma_c}{100\times10^3}$$

$$\frac{2.25 \sigma_c}{200\times10^3} + \frac{\sigma_c}{100\times10^3} = 9\times10^{-4} - 6\times10^{-4}$$

$$1.125\times10^{-5} \sigma_c + 1\times10^{-5} \sigma_c \quad \cancel{\frac{2.25\sigma_c + 2\sigma_c}{200\times10^3}} = 3\times10^{-4} \; \cancel{\times 200}$$

$$\cancel{2.25\sigma_c = 160}$$

$$2.125\times10^{-5} \sigma_c = 3\times10^{-4}$$

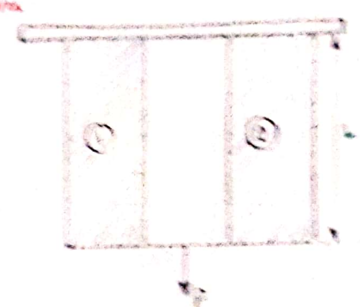
$$\boxed{\sigma_c = 14.117 \text{ N/mm}^2} \quad \text{——— (2)}$$

Substituting (2) in (1) $\sigma_s = 2.25 \times 14.117$

$$\boxed{\sigma_s = 31.76 \text{ N/mm}^2} \quad \text{——— (1)}$$

Analysis of Bars of Composite Sections

A bar made up of two or more bars of equal length but of different materials rigidly fixed so as to behave as one unit when subjected to axial loads is called a composite bar.



1. Extension or compression in each bar is equal. Hence strain in each bar is equal.

2. Total external load = Sum of loads carried by different material

$\therefore P = P_1 + P_2 + \ldots$ ——— (1)

Stress in bar 1, $\sigma_1 = \frac{P_1}{A_1}$ (or) $P_1 = \sigma_1 A_1$ ——— (2)

Illy Stress in bar 2, $\sigma_2 = \frac{P_2}{A_2}$ (or) $P_2 = \sigma_2 A_2$ ——— (3)

Hence $P = \sigma_1 A_1 + \sigma_2 A_2$ ——— (4)

Strain in bar 1, $e_1 = \frac{\sigma_1}{E_1}$ ——— (5)

Illy strain in bar 2, $e_2 = \frac{\sigma_2}{E_2}$ ——— (6)

Since, $e_1 = e_2$

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

Modular ratio: The ratio of $\frac{E_1}{E_2}$ is called modular ratio ie the ratio of Youngs modulus of material 1 to Youngs modulus of material 2.

UNIT - II

Transverse loading on beams and stresses in beam

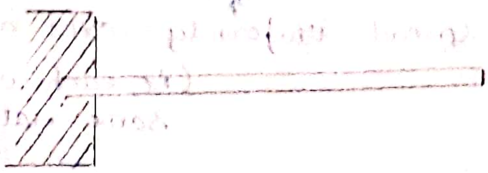
Forces applied ⊥r to the longitudinal axis of member. The loading causes member to bend and deflect from its original position with internal tensile & compressive strains accompanying change in curvature

Beams

Types of beams

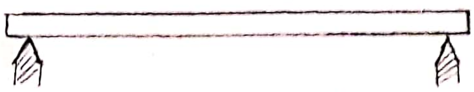
Cantilever Beam.

- A beam which is fixed at one end and free at the other end.



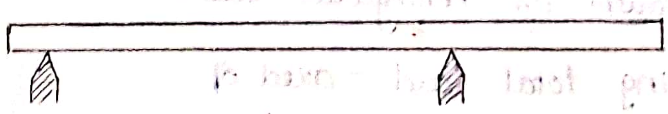
Simply supported Beam

- A beam resting freely on the supports at its both ends.



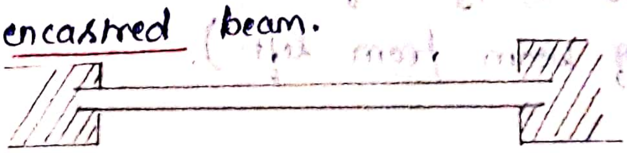
Overhanging Beam

- A beam in which any of the end portion is extended beyond the support.



Fixed Beams

- A beam whose both ends are fixed or built in. Also known as built-in or encastred beam.



Continuous Beam

- A beam which is provided more than two supports.

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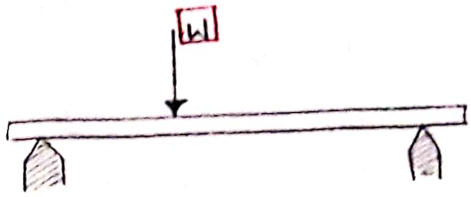
Types of load

- Normally beams are horizontal and loads are vertical.

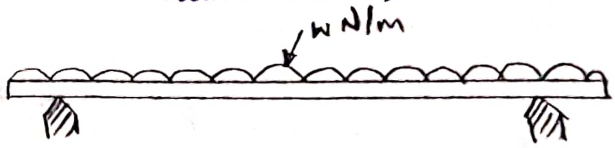
Concentrated or point load



- A load which acts at a point
- (in practice it is distributed over a small area).

Uniformly Distributed load (rate of loading w is uniform along the length).

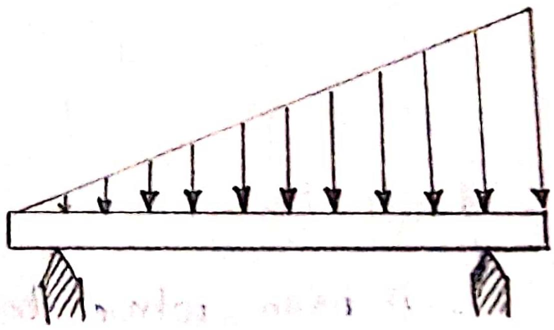
- A load which is spread uniformly over a beam. (i.e. each unit length loaded at same rate).



- Expression : w N/m
- Representation : u.d.l.
- (for solving total u.d.l is converted into point load, acting at centre)

Uniformly Varying load

- The load is spread over a beam in such a manner that the rate of loading varies from point to point.
- Also known as triangular load.
- (for solving total load = area of triangle



*This load is assumed to act at C.G of Δle i.e, 2/3rd of total length of beam from left).

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Shear force

- The algebraic sum of vertical forces at any section of the beam to the right or left of the section is known as shear force.
- written as S.F

Shear force diagram

- It is the one which shows the variation of the shear force along the length of the beam.

Bending Moment

- The algebraic sum of the moments of all the forces acting to the right or left of the section is known as bending moment.

Bending moment diagram

- The diagram which shows the variation of bending moment along the length of the beam.

Sign Conventions for Shear Force and Bending Moment

Shear force

- Resultant of the load & reaction to the right of X-X is 500 N downwards (1000↓ − 500 ↑ = 500↓).

- Resultant force acting on any one of the parts normal to the axis of the beam is called shear force. (Here it is 500 N Shear force)

- Shear force at a section is (+ve) when resultant of forces to the left of section is (↑) upward, (or) right of section is ↓.

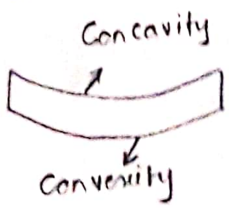
- Illy shear force at a section is (−ve) if resultant of forces to the left of section is ↓ (or) to right of section is ↑.

What S is here? Shear force (+ve) or (−ve)

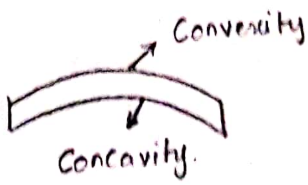
Bending Moment

- B.M at a section is considered (+ve) if B.M at that section tends to bend the beam to a curvature having concavity at top.

- Illy B.M at a section is (−ve) if B.M at that section tends to bend the beam to a curvature having convexity at top.



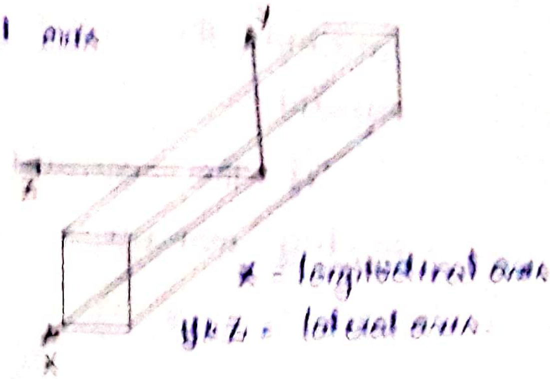
+ve B.M (or) Sagging.M

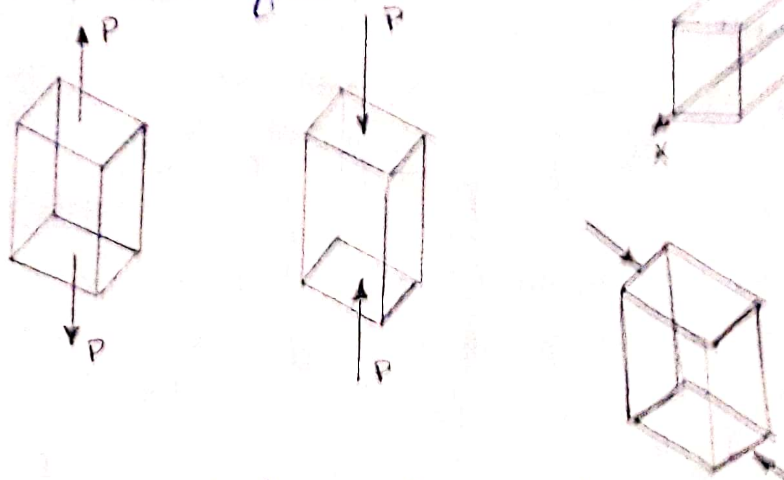


−ve B.M (or) hogging.M

Force: Force is push or pull.

- Force applied in the longitudinal axis tends the member to elongate (Tensile force) or compress (Compressive force)





- Force applied in the lateral axis try to slice of the member (shear) or would try to bend the member (Bending moment.)

[Amount of elongation, compression (or) shearing is directly dependent on magnitude of force].

↓

[- This case is not the same with rotation
- The same amount of force if applied at greater distance would produce greater rotation.]

- The turning effect of a force is known as moment (Nm)
- Moment of force is product of force and distance of force from the point of interest.

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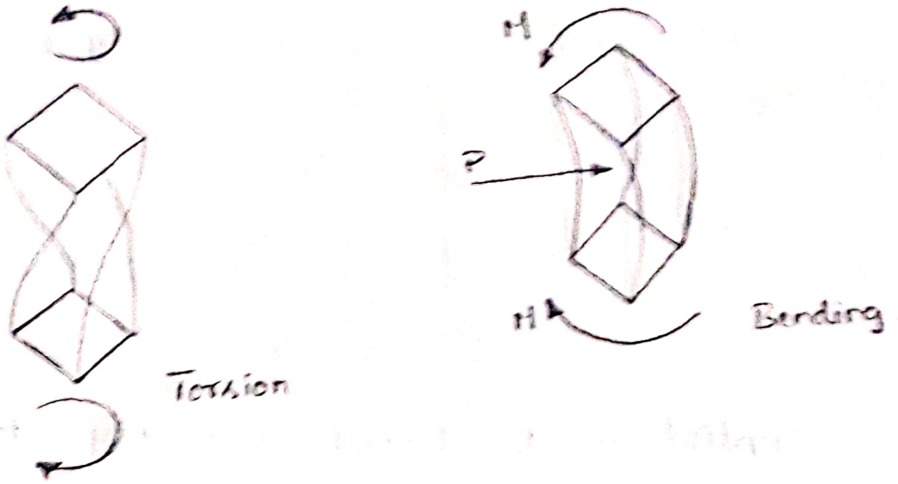
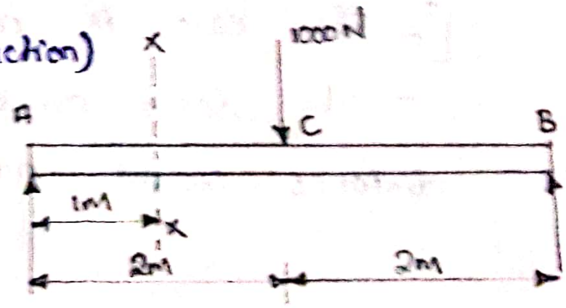
- If the moment of force tries to twist the member then ~~we call~~ it is twisting moment or torsional moment.

- If the moment tries to bend the member it is bending moment.



Consider the simply supported beam AB, carrying a load of 1000 N at its middle point. Reactions R_A and R_B are equal and are having magnitude 500 N. Imagine beam to be divided into two portions by the section X-X. Let the section X-X is at a distance of 1m from A.



- The moment of all forces (load & reaction) to the left of section X-X

$= R_A \times 1 = 500 \times 1 = 500\,Nm\ (CW).$

- IIIly moment of all forces to the right of the section

$= (R_B \times 3\ (CCW)) - (1000 \times 1\ (CW))$

$= (500 \times 3) - 1000$

$= 500\ Nm\ (CCW)$

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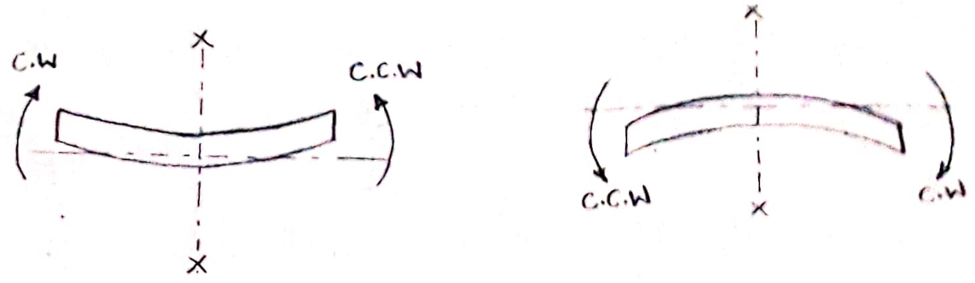

- The bending moment at a section is the algebric sum of moments of forces and reactions acting on one side of the section. X-X is 500 Nm.

- <u>Conditions</u>

- B.M is considered (+ve) when the moment of forces & reaction on the left portion is C.W. & to the right of the portion is C.C.W.

- Illy B.M is considered (-ve) when the moment of forces & reaction on the right of portion is C.W & on the left portion is C.C.W.

Important Points for drawing Shear Force and Bending Moment Diagram.

1. Consider left or right portion of the section.
2. Add the forces (including reaction) normal to the beam on one of the portion.

(*If right portion of the section is chosen, force acting downwards is positive while force acting upwards is -ve.

* If left portion of the section is chosen, force acting upwards is +ve, while force acting downwards is -ve).

3. +ve values of shear fore and B.M are plotted above the base line, -ve values are plotted below the base line.

4. The S.F dig. will increase or decrease suddenly i.e., by a vertical straight line at a section where there is vertical pt. load.

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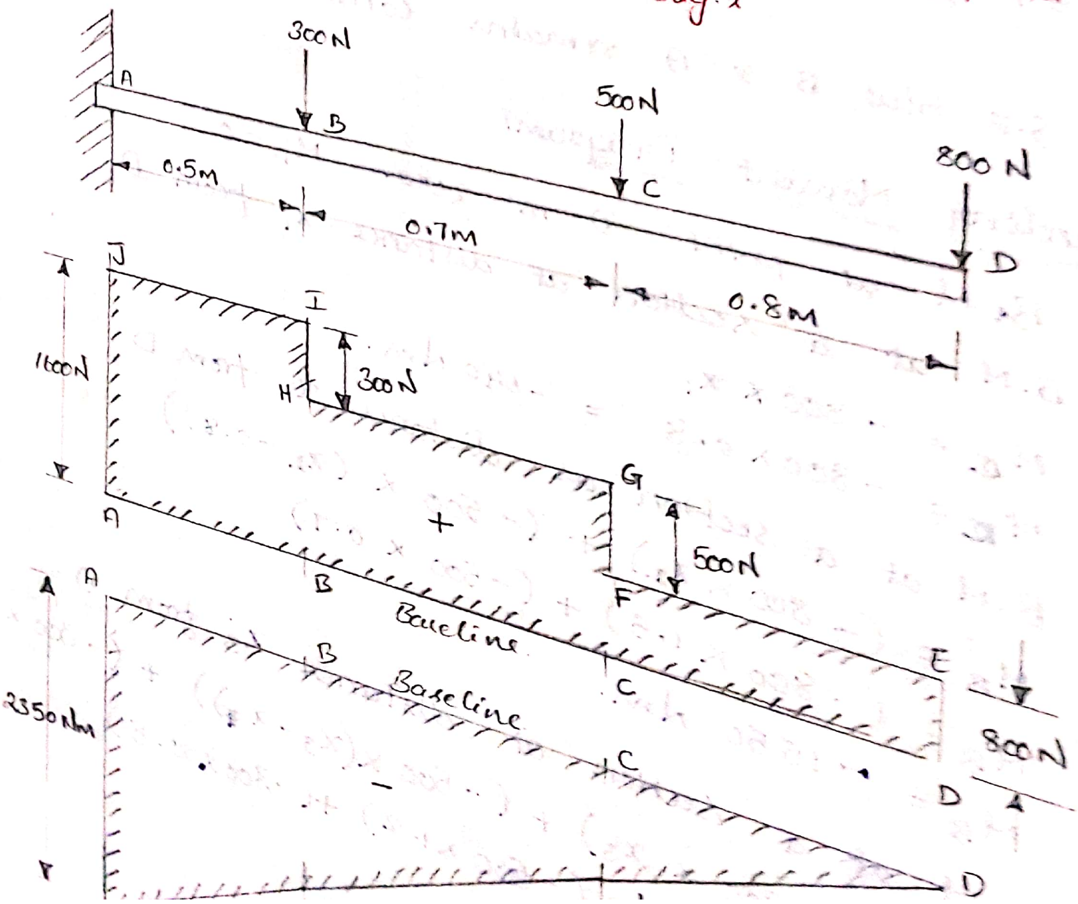
5. The shear force b/w any two vertical loads will be constant and hence the S.F diagram b/w two vertical loads will be horizontal.

6. The B.Ms at the two supports of S.S.B and at the free end of a Cantilever will be zero.

Problem 1:

A Cantilever beam of length 2m carries the point loads as shown. Draw the S.F and B.M dig.s



Solution :

S.F Diagram.

* Shear force at D = +800 N
* Shear force remains constant b/w D & C.
* Shear force at C due to point load = 800 + 500 = +1300 N

* S.F b/w C & B remains constant.
* S.F at B due to point load = 800 + 500 + 300 = 1600 N.
* S.F b/w B & A remains constant.

Bending Moment Diagram:

* B.M at point D is zero. $M_D = 0$
* B.M at a section at distance x_1 from D

$M_C = -800 \times x_1$

$M_C = -800 \times 0.8 = -640$ Nm.

* B.M at a section at distance x_2 from D

$M_B = (-800 \times x_2) + (-500 \times (x_2 - 0.8))$

$M_B = (-800 \times 1.5) + (-500 \times 0.7)$

$M_B = -1550$ Nm.

* B.M at a section at distance x_3 from D.

$M_A = (-800 \times x_3) + (-500 \times (x_3 - x_1)) + (-300 \times (x_3 - x_2))$

$M_A = (-800 \times 2) + (-500 \times 1.2) + (-300 \times 0.5)$

$M_A = -2350$ Nm.

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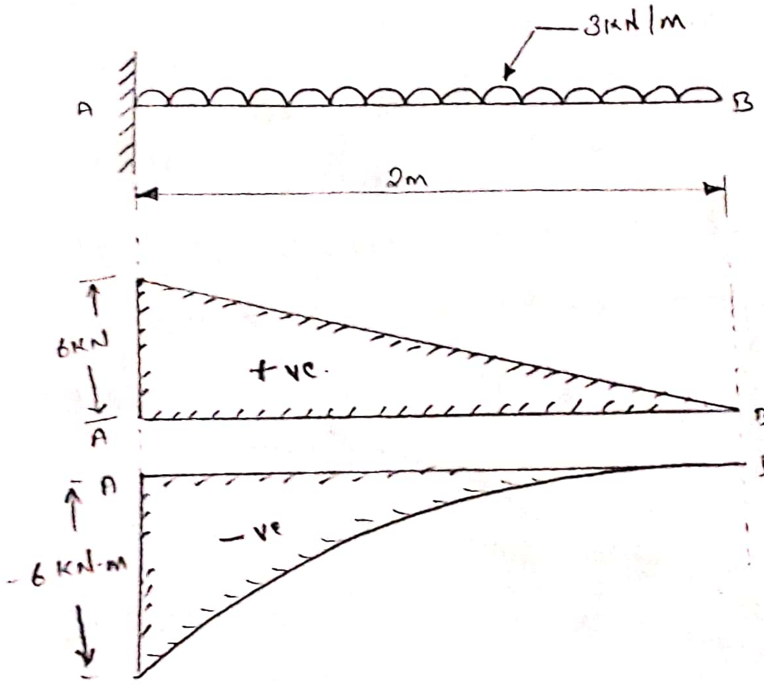
② A Cantilever beam of length 2m carries the point load of 1kN at its free end, and another load of 2kN at a distance of 1m from the free end. Draw SF and BM dig. for cantilever beam.

③ A Cantilever beam of length 2m carries a UDL of 3kN/m. Draw SF and BM Diagrams.

Solution:



SF Calculation

$SF_B = 3 \times 0 = 0$

$SF_A = 3 \times 2 \boxed{= 6kN}$

B.M Calculation

$BM_B = (-3\times2) \times \frac{0}{2} = 0$

$BM_A = (-3\times2) \times 2/2$

$\boxed{= -6 kN m.}$

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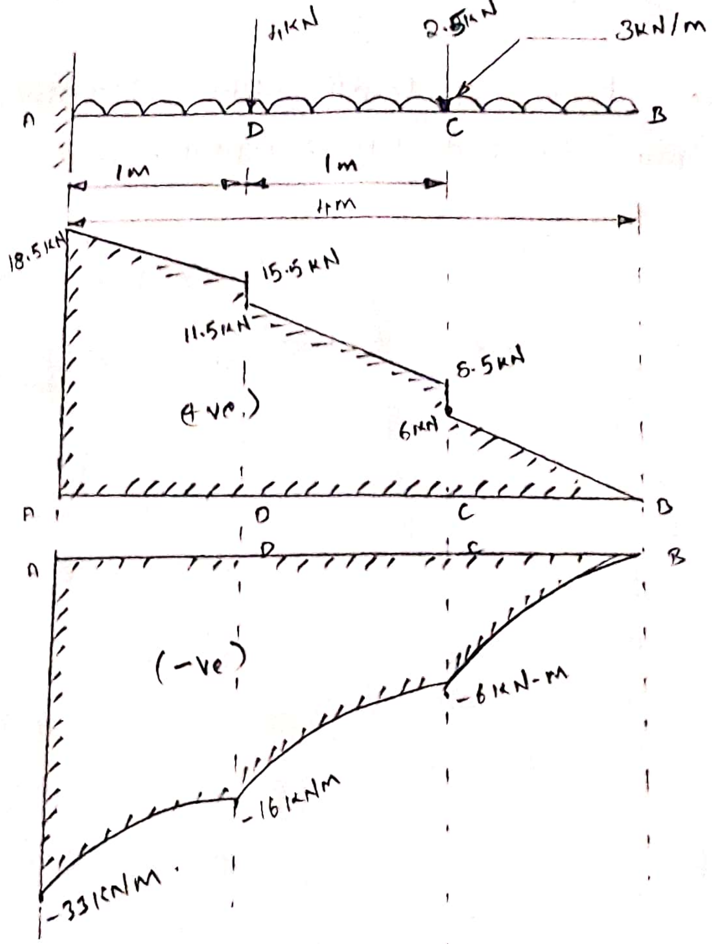
(4) A Cantilever of length 4m carried UDL of 3 kN/m run over the whole length and two point loads of 4 kN and 2.5 kN are placed 1m and 2m respectively from the fixed end. Draw the SF and BM diagrams.



Shear Force Calculation:

$SF_B = 0$

SF_C = SF due to point load of 2.5kN at C (+)
udl b/w B & C

$= 2.5 + (3 \times 2)$

$= 2.5 + 6$

$= 8.5$ kN

SF_D = S.F due to pl. load at C & D (+)
S.F due to U.d.l b/w D & B

$= 2.5 + 4 + (3 \times 3)$
$= 6.5 + 9$
$= 15.5$ KN

$S.F_A$ = S.F due to Pl load at D & C (+)
S.F due to U.d.l b/w A & B

$= 2.5 + 4 + (3 \times 4)$
$= 6.5 + 12$
$= 18.5$ KN.

Bending moment Calculations.

$BM_B = (-3 \times 0) \times 0/2$
$= 0$

$BM_C = (-2.5 \times 0) + (-3 \times 2) \times 2/2$
$= 0 - 6$
$= -6$ KN-m.

$BM_D = (-3 \times 3) \times 3/2 + (-2.5 \times 1) + (-4 \times 0)$
$= -13.5 \cancel{+11.23} - 2.5$
$= -16$ KN.m

$BM_A = (-3 \times 4) \times 4/2 + (-2.5 \times 2) + (-4 \times 1)$
$= -24 - 5 - 4$
$= -33$ KN.m.

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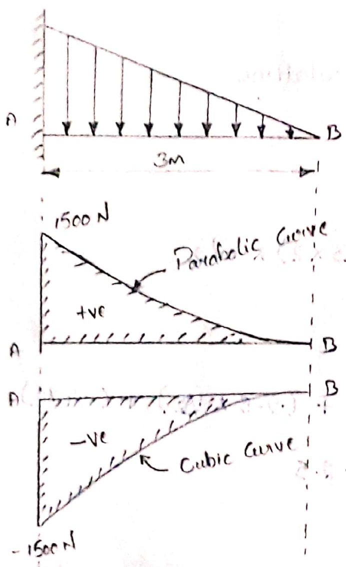
A Cantilever of 3m long carries a UDL of 12 kN/m is spread over a length of 1.5 m from the free end. It also carries a point load of 15kN at free end and another point load of 8kN at 1m from the fixed end. Draw the SFD and BMD

A Cantilever beam of 3m long carries a gradually varying load, zero at the free end to 1000 N/m at the fixed end. Draw the SF and BM diagrams for the beam.



<u>Calculation of S.F's.</u>

$SF_B = 0$

SF_A = Area of triangle ABO

$= \frac{1}{2} \times 3 \times 1000$

$= 1500$ N

<u>Calculation of B.M.</u>

$BM_B = 0$

BM_A = Load equivalent to pt load × Distance of load from A.

$= (-1000 \times 3 \times \frac{1}{2}) \times (\frac{1}{3} \times 3)$

$= -1500$ Nm.

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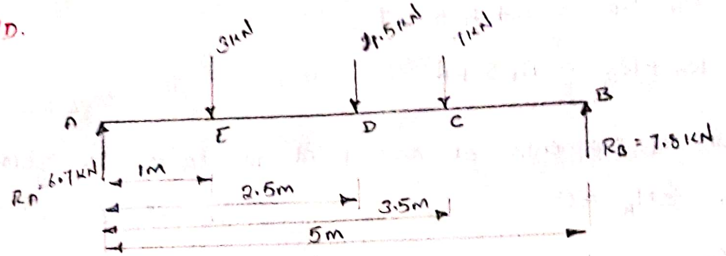
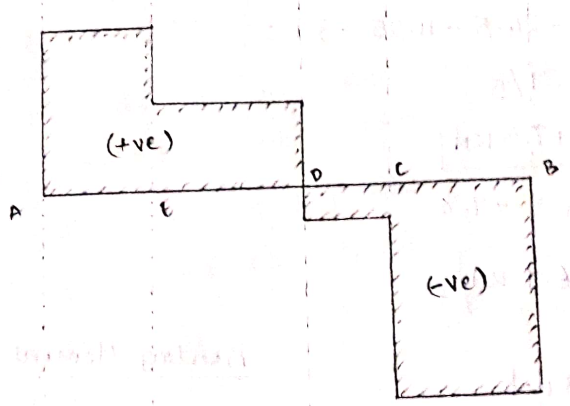
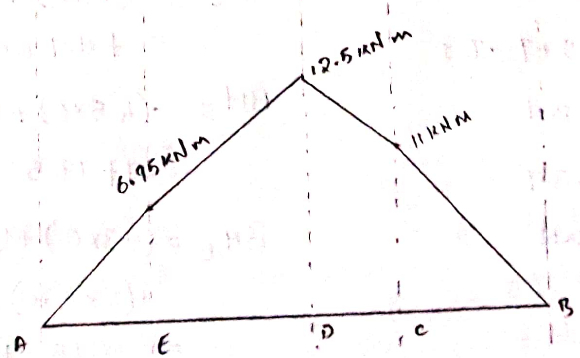
Problems on Simply Supported Beams

① A beam freely supported over an effective span of 5 m carries pt loads 3KN, 4.5KN and 1KN at 1, 2.5 and 3.5 m respectively from the left hand support. Construct S.F.D and B.M.D.







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Calculation of Shear Force.

First Calculate reaction forces. R_A & R_B.

Assumption: the Beam should be in equilibrium.

i.e., Sum of Upward forces = Sum of downward forces.

$R_A + R_B = 3 + 4.5 + 7$

$R_A + R_B = 14.5$ kN

Sum of moments at any point is equal to zero.

i.e. $\Sigma M_A = 0$

$(R_B \times 5) + (-7 \times 3.5) + (-4.5 \times 2.5) + (-3 \times 1) = 0$

$5R_B - 24.5 - 11.25 - 3 = 0$

$R_B = 39/5$

$\boxed{R_B = +7.8 \text{ kN}}$

$R_A = 14.5 - 7.8$

$\boxed{R_A = 6.7 \text{ kN}}$

$SF_B = -7.8$ kN

$SF_C = +7 - 7.8$

$= -0.8$

$SF_D = +4.5 + 7 - 7.8$

$= 3.7$ kN

$SF_E = 3 + 3.7$

$= 6.7$ kN.

Bending Moment Calculations.

$BM_B = -7.8 \times 0 = 0$

$BM_C = (7 \times 0) + (7.8 \times 1.5)$

$= +11.7$ kN-m.

$BM_D = -(4.5 \times 0) + (-7 \times 1) + (7.8 \times 2.5)$

$= -7 + 19.5 = 12.5$ kN m.

$BM_E = (-3 \times 0) + (-4.5 \times 1.5) + (-7 \times 2.5) + (7.8 \times 4)$

$= -6.75 - 17.5 + 31.2$

$= 6.95$.

$BM_A = 0$.

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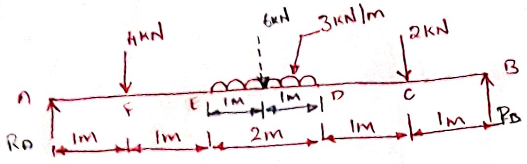
SFD and BMD for simply supported beam



Total udl value = 3x2
= 6 KN
(i.e., for 1 m it is 3KN
as udl occupies 2 meter
udl = 3x2 = 6 KN)

STEP : Calculate the reaction forces R_A & R_B

Assumption : The Beam should be in equilibrium.

i.e., Sum. of Upward forces = Sum of downward forces.

$R_A + R_B = 2+6+4$

$R_A + R_B = 12$ KN

• Sum of moments at any point is equal to zero.

i.e, $\overset{\curvearrowright}{\Sigma M_A} = 0$ Moment about pt A.

[Consider right portion of any section to calculate S.F]

$(R_B \times 6) - (2\times5) - (6\times3) - (4\times1) = 0$

$6R_B = 10 + 18 + 4$

$R_B = 32$

$\boxed{R_B = 5.34 \text{ KN}}$

$R_A = 12 - 5.34$

$\boxed{R_A = 6.66 \text{ KN}}$

Calculation of Shear force :

$SF_B = -5.34$ KN

$SF_C = -5.34 + 2$
$= -3.34$ KN

$SF_D = -5.34 + 2$
$= -3.34$ KN

$SF_E = -3.34 + 6$
$= 2.66$ KN

$SF_F = 2.66 + 4$
$= 6.66$ KN

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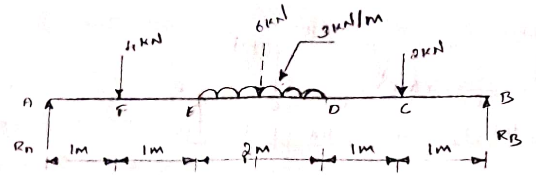
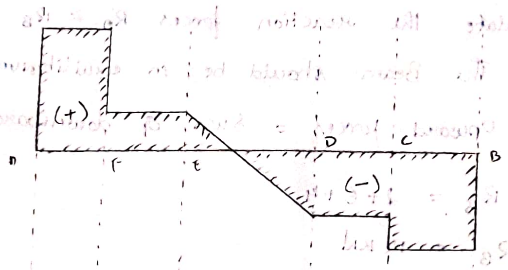
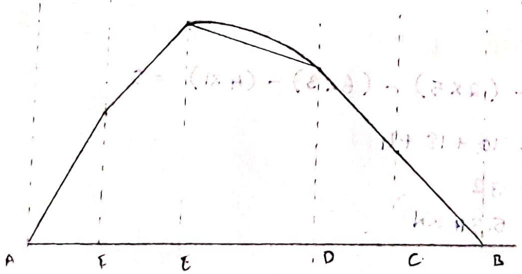




~~Calculation of Bending Moment~~



Calculation of Bending moment (B.M)

$BM_B = 5.34 \times 0$

$= 0$ kN.m

$BM_C = (5.34 \times 1) + (2 \times 0)$

$= 5.34$ kNm

$BM_D = (5.34 \times 2) - (2 \times 1)$

$= 8.68$ kNm

$BM_E = (5.34 \times 4) - (2 \times 3) - (6 \times 1)$

$= 9.36$ kNm

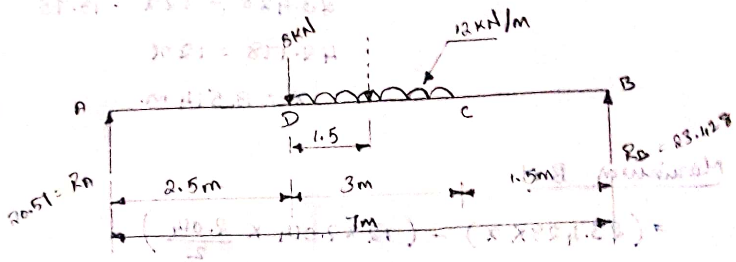
$BM_F = (5.34 \times 5) - (2 \times 4) - (6 \times 2)$

$= 6.7$ kNm

$BM_A = (5.34 \times 6) - (2 \times 5) - (6 \times 3) - (4 \times 1)$

$= 0$

A simply supported beam of 7m span has a load of 12 kN/m uniformly distributed over 3m. It is 1.5m away from the right. In addition it has a point load of 8kN at 2.5m from the left hand support. Find the value of maximum bending moment.



Calculation of Reaction Forces:

$R_A + R_B = 8 + (12 \times 3)$

$= 44 \text{ kN}$

$\Sigma M = 0$

$(R_B \times 7) + (-12 \times 3 \times (3/2 + 2.5)) + (-8 \times 2.5) = 0$

$7R_B - 144 - 20 = 0$

$7R_B = 164$

$\boxed{R_B = 23.428 \text{ kN}}$

$R_A = 44 - 23.428$

$\boxed{R_A = 20.57 \text{ kN}}$

Calculation of S.F

$SF_B = -23.428 \text{ kN}$

$SF_C = -23.428 \text{ kN}$

$SF_D = \boxed{-23.428 + (12 \times 3)} + 8$

$= 12.572 + 8$

$= 20.57 \text{ kN}$

Calculation of BM.

$BM_B = 0$

$BM_C = 23.428 \times 1.5 = 35.142 \text{ kNm}.$

$BM_D = (23.428 \times 4.5) - (12 \times 3 \times 3/2)$

$= 51.126 \text{ kNm}$

$BM_A = (23.428 \times 7) - (12 \times 3 \times (3/2 + 2.5))$
$- (8 \times 2.5)$

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$SF_A = +R_A = 20.572\ kN$

$BM_A = 163.996 - 135 - 20$
$= 0$

The max. B.M is at distance x from pt B, where S.F changes sign.

$0 = -23.428 + (12x(x - 1.5))$

$23.428 = 12x - 18.75$

$42.178 = 12x$

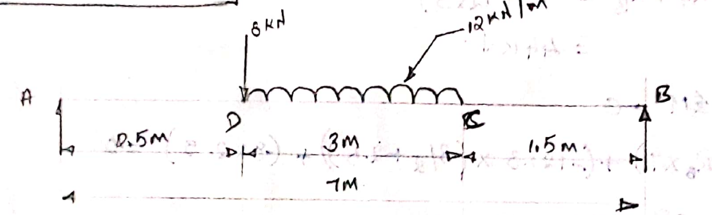
$\boxed{x = 3.514\ m.}$

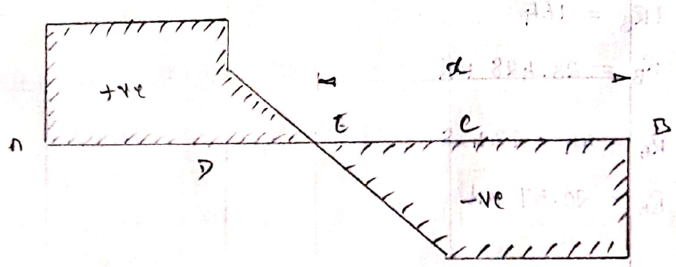
<u>Maximum B.M</u>

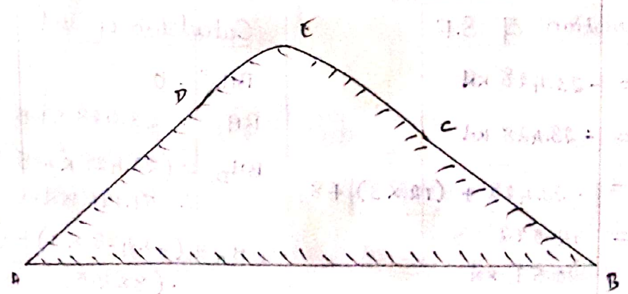
$= (23.428 \times x) - \left(12 \times 2.014 \times \frac{2.014}{2}\right)$

$= 82.325 - 24.337$

$\boxed{= 57.988\ kN\text{-}m.}$







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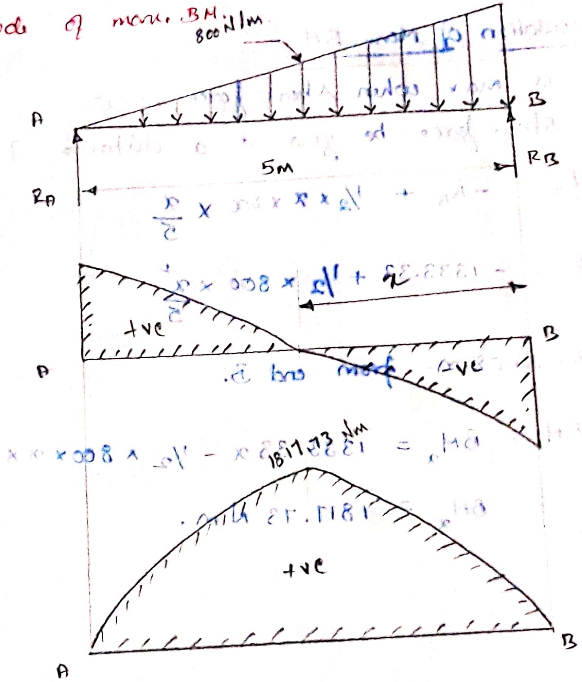
A Simply supported beam of length 5m carries a uniformly varying load of 800 N/m run at one end to zero at other end. Draw SF & BM diagrams. Also calculate the position and magnitude of max. BM.



Calculation of reaction Forces :

$R_A + R_B = \frac{1}{2} \times 5 \times 800$

$R_A + R_B = 2000\ N$

$\boxed{R_A = 2000 - R_B}$

$R_A = 2000 - 1333.33$

$\boxed{R_A = 666.66\ N}$

$\Sigma B_A = 0$

$R_B \times 5 - (\frac{1}{2} \times 5 \times 800 \times \frac{2}{3} \times 5) = 0$

$5R_B = 6666.66$

$\boxed{R_B = 1333.33\ N}$

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~~Calculation of SF~~

Calculation of BM :

$BM_B = 0$

$BM_A = 0$

Calculation of Max. BM

BM is max. when shear force is 0.

let shear force be zero at a distance of x from B.

$$SF_x = -R_B + \frac{1}{2} \times x \times 800 \times \frac{x}{5}$$

$$0 = -1333.33 + \frac{1}{2} \times 800 \times \frac{x^2}{5}$$

$x = 4.08$ m. from end B.

$$\text{Max. BM} = BM_x = 1333.33x - \frac{1}{2} \times 800 \times x \times x/5 \times 2/3 \times x.$$

$$BM_x = 1817.73 \text{ N m.}$$

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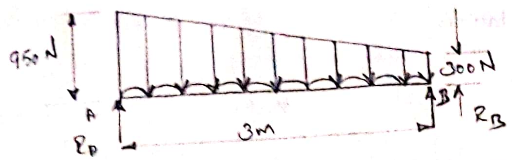
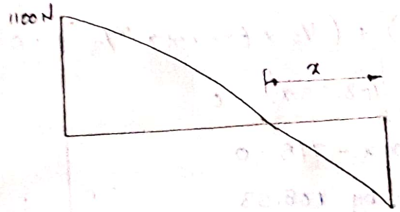
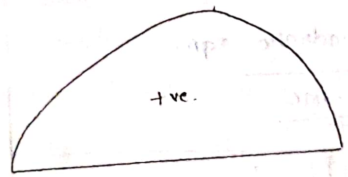
Draw the SF and BM ~~for the~~ diagram for the given beam.







Calculation of reaction forces

$R_A + R_B = (300 \times 3) + (1/2 \times (950 - 300) \times 3)$

$= 900 + 975$

$R_A + R_B = 1875$

$\Sigma M_A = 0$

$R_B \times 3 - 300 \times 3 \times 3/2 - 650 \times 1/3 \times 3 \times 1/2 \times 3 = 0$

$3R_B = 975 + 1350$

$\boxed{R_B = 775 \text{ N}}$

$R_A = 1875 - 775$

$\boxed{R_A = 1100 \text{ N}}$

SF Calculation

$SF_B = -R_B = -775\,N$

$SF_A = R_A = 1100\,N$

BM Calculation

$BM_B = 0$

$BM_A = 0$

Calculation of Max. BM.

$-R_B + (300 \times x) + (1/2 \times 650 \times x \times x/3) = 0$

$-775 + 300x + 108.33x^2 = 0$

$108.33x^2 + 300x - 775 = 0$

Dividing the eqn by 108.33

$x^2 + 2.77x - 7.154 = 0$

Solving the quadratic eqn.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2.77 \pm \sqrt{2.77^2 - 4(1)(-7.154)}}{2}$$

$$= \frac{-2.77 \pm \sqrt{7.67 + 28.61}}{2}$$

$$= \frac{-2.77 \pm 6.023}{2}$$

$\boxed{x = 1.626m}$ $\quad x = -4.3965$

Max. BM = $BM_x = R_B \times x - 300 \times x \times x/2 - 1/2 \times x \times \frac{650 \times x}{3} \times 2/3 \times x.$

$= 775x - 150x^2 - \frac{650}{9}x^3$

$= 552.8\,Nm.$

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THEORY OF SIMPLE BENDING

PURE BENDING (OR) SIMPLE BENDING

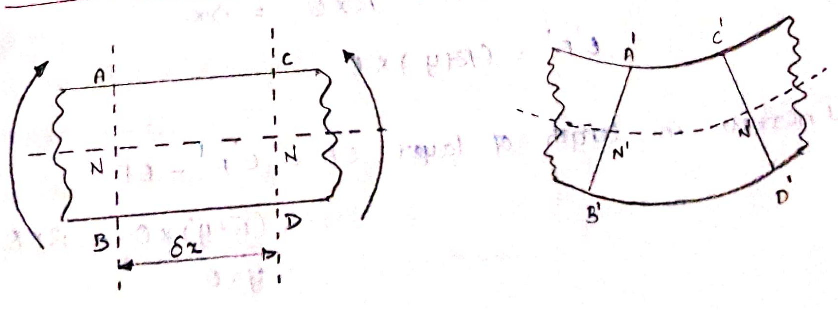
If a length of a beam is subjected to constant bending moment and no shear force (i.e., zero) then the stresses will be setup in that length due to B.M only and is called pure bending. The stresses setup is known as bending stresses.

Homogeneous: same kind of material throughout | Isotropic: elastic properties in all directions are equal.

Assumptions made in theory of Simple Bending

* The beam material is homogeneous and isotropic
* Value of youngs modulus of elasticity is same in tension and compression.
* transverse section which were plane before bending, remain plane after bending.
* The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
* The radius of curvature is large compared to dimensions of cross sections.
* Each layer of beam is free to expand (or) contract, independently of the layer, above or below it.

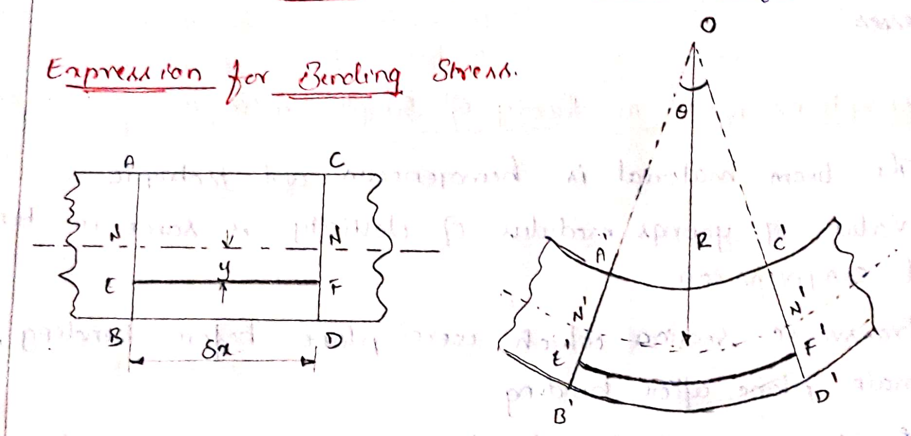
Theory of Simple bending:



* Due to the BM, all the layers of the beam, which were originally of the same length, do not remain of the same length.

* At a level b/w the top and bottom of the beam, there will be a layer which is neither shortened nor elongated. This laye is neutral layer or neutral surface.

Expression for Bending Stress.



R = Radius of neutral layer $N'N'$ from centre of curvature.

θ = Angle subtented b. at O by $A'B'$ & $C'D'$

Strain variation along depth of beam.

* Consider layer EF at distance y from NN.

Original length of layer $EF = NN = \delta x$.

After bending $N'N' = NN = R \times \theta = \delta x$.

$$E'F' = (R+y) \times \theta$$

Increase in length of layer $EF = E'F' - EF$

$$= (R+y) \times \theta - R \times \theta$$

$$= y \times \theta$$

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Strain in layer EF = $\frac{\text{Increase in length}}{\text{Original length}}$

$= \frac{y \times \theta}{R \times \theta}$

$= y/R$

R is constant ∴ strain is proportional to distance of layer from neutral axis.

Stress variation.

σ = Stress in layer EF

E = youngs modulus

$E = \frac{\text{stress in layer EF}}{\text{strain in layer EF}}$

$E = \frac{\sigma}{y/R}$

$\sigma = E \times y/R$

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}}$$

Neutral axis and moment of resistance

Moment of Resistance

$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

M in Nm ; I in m^4

σ in N/m^2 ; y in m

E in N/m^2 ; R in m

A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine max. stress induced and BM which will produce max. stress. Take $E = 2\times10^5\ N/mm^2$

Given:

width, (b) = 120 mm

thickness, (t) = 20 mm.

youngs modulus $E = 2\times10^5\ N/mm^2$

radius of curvature $R = 10$ m

$= 10\times10^3$ mm.

Solution:

Moment of Inertia, $I = \frac{bt^3}{12} = \frac{120 \times 20^3}{12} = 8 \times 10^4 \ mm^4$

Maximum stress will occur either at the topmost layer or bottom most layer.

$\therefore y = t/2 = 20/2 = 10 \ mm.$

$\frac{\sigma}{y} = \frac{E}{R}$ (or) $\sigma = \frac{E}{R} \times y$

$\sigma = \frac{2 \times 10^5}{10 \times 10^3} \times 10$

$\boxed{\sigma = 200 \ N/mm^2}$

B.M which creates maximum stress is

$\frac{M}{I} = \frac{E}{R}$

$M = \frac{E}{R} \times I$

$M = \frac{2 \times 10^5}{10 \times 10^3} \times 8 \times 10^4$

$\boxed{M = 1.6 kN\text{-}m.}$

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A beam of 6m long rests on supports 5m apart, the right hand end is overhanging by 1m. The beam carries a U.D.L of 20 kN/m over the entire length of beam. Draw SFD & BMD indicating max. BM and the pt. of contraflexure.

$R_B = 72$ kN.
$R_A = 48$ kN.

$SF_C = 0$
SF_B (without R_B) $= 20$ kN
SF_B (with R_B) $= -52$ kN
$SF_A = R_A = 48$ kN

$BM_C = 0$
$BM_B = -10$
$BM_A = 0$

Max. BM.

$SF_x = 20x - 72 = 0$

$x = 3.6$ m.

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UNIT - III

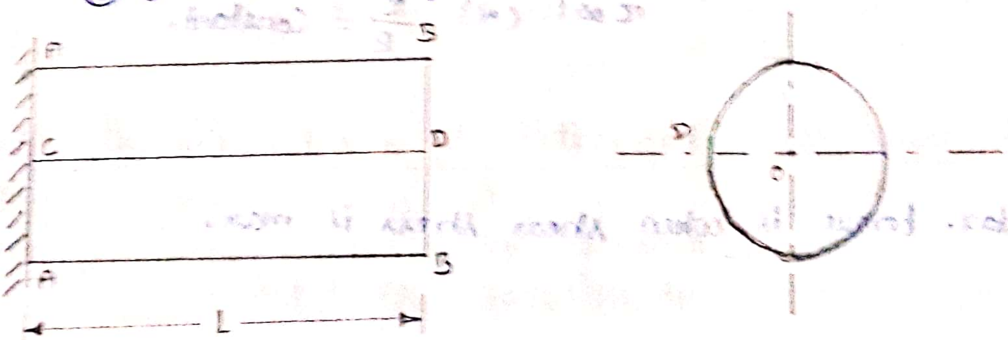
TORSION

* A shaft is said to be in torsion, when equal & opposite torques are applied at the two ends of the shaft.
* Torque = tangential force applied to ends of shaft × radius of the shaft.
* Due to torque applied, shaft is subjected to twisting moment.
* This induces shear stress & shear strain.

Derivation of Shear Stress produced in a Circular Shaft subjected to torsion

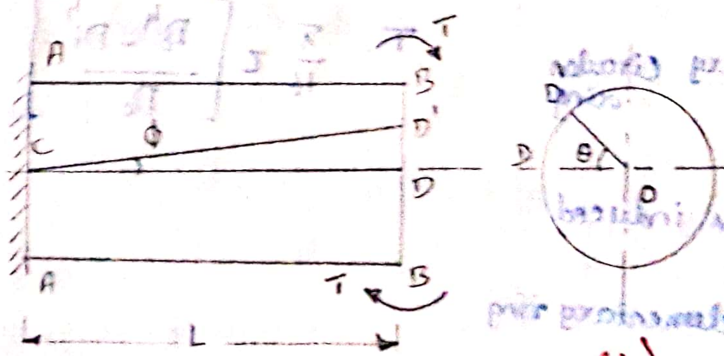
Determination: Magnitude of shear stress at any point on the shaft.

Consideration: ① Shaft fixed at end AA.
② Free at end BB.
③ Line CD on outer surface of shaft.
④ Subjected to torque T at end BB.
⑤ point D will shift to D'. Hence CD is deflected to CD'.
⑥ Line CD ~~was~~ shifted to CD'.



R = Radius of shaft
L = Length of shaft
C = Modulus of rigidity of material

T = Torque applied at end BB
τ = Shear stress induced at surface
ϕ = ∠DCD' also equal to shear strain.
θ = ∠DOD' (called angle of twist).



(a)

(b)

From (a)

Distortion at outer surface due to $T = DD'$

$\therefore$ Shear strain = Distortion per unit length

$$= \frac{DD'}{L} = \frac{DD'}{CD} = \tan\phi = \phi \quad \left[\because \tan\phi \text{ is small then } \tan\phi \approx \phi \right]$$

— ①

From (b)

$DD' = CD \times \theta = R\theta$ — ②

Substituting ② in ①

$$\phi = \frac{R \times \theta}{L}$$

Modulus of rigidity $(C) = \dfrac{\text{Shear stress}}{\text{Shear strain}}$

$$C = \frac{\tau}{\left(\frac{R\theta}{L}\right)} = \frac{\tau \times L}{R\theta}$$

$$\frac{C\theta}{L} = \frac{\tau}{R}$$

C, θ, L are constant.

$$\boxed{\tau = \frac{R \times C \times \theta}{L}}$$

$\tau \propto R$ (or) $\dfrac{\tau}{R}$ = Constant.

Maximum torque transmitted by a solid circular shaft.

Max. torque is when shear stress is max.

Refer R.K. Bansan 16.3 (pg: 681) for derivation

Total torque $(T) = \dfrac{\pi}{16} \tau D^3$

Torque transmitted by hollow circular shaft

R_o = outer radius

R_i = Inner radius.

r = Radius of elementary circular ring

dr = thickness of ring

τ = max. shear stress induced at outer surface

q = Shear stress on elementary ring

dA = Area of elementary ring $= 2\pi r \times dr$

Derivation: 16.4

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$$

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Power transmitted by shafts

Power $= \frac{2\pi NT}{60}$ watts

$= \omega \times T$

N = r.p.m of shaft.
T = Mean torque transmitted N-m.
ω = Angular speed.

Problem 1:

A solid shaft of 150mm diameter is used to transmit torque. Find the max. torque transmitted by shaft if max. shear stress is 45 N/mm².

Given:

Dia, D = 150mm

Max. shear stress (τ) = 45 N/mm²

To find:

Max. torque transmitted T

Solution:

$T = \pi/16 \times 45 \times 150^3$

$\boxed{T = 29820586 \text{ N.mm}}$

H.W **Problem 2:**

The shearing stress of a solid shaft is not to exceed 20 N/mm² when the torque transmitted is 10000 Nm. Determine the minimum dia of shaft.

Problem 3:

In a hollow circular shaft of outer & inner dia of 20 cm and 10cm respectively, the shear stresses is not to exceed 40 N/mm². Find the max. torque which the shaft can easily transmit.

Given:

Outer dia (D_o) = 20 cm
= 200 mm

Inner dia (D_i) = 10cm
= 100 mm

Max. shear stress (τ) = 40 N/mm².

To find:

Max. Torque transmitted (T).

Solution:

$$T = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o}\right] = \frac{\pi}{16} \times 40 \left[\frac{200^4 - 100^4}{200}\right]$$

$\boxed{T = 58904860 \text{ N-mm}}$.

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Problem 4:

Two shafts of the same material and of same length are subjected to the same torque, if the first shaft is of a solid circular section and second shaft is of hollow circular section, whose internal dia is 2/3 of the outside dia and max. shear stress developed in each shaft is the same, compare the weights of the shaft.

Given

T = torque transmitted by both shafts

τ = Max. shear stress in both shafts.

D = outer dia of solid shaft.

D_o = outer dia of Hollow shafts

D_i = Inner dia of hollow shaft.

W_s = weight of solid shaft

W_h = weight of hollow shaft.

L = length of both shaft.

w = weight density of each shaft material.

Solution:

Torque transmitted by solid shaft, $(T) = \frac{\pi}{16} \tau D^3$

Torque transmitted by hollow shaft $(T) = \frac{\pi}{16} \tau \left[\frac{D_o^4 - D_i^4}{D_o} \right]$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - (2/3\, D_o)^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \left[\frac{D_o^4 - \frac{16}{81} D_o^4}{D_o} \right]$$

$$= \frac{\pi}{16} \tau \times \frac{65 D_o^3}{81}$$

As torque transmitted by shafts are equal

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \times \frac{65 D_o^3}{81}$$

$$D^3 = \frac{65}{81} D_o^3$$

③

Weight of solid shaft, W_s = weight Density × Volume of solid shaft

$= w \times$ Area of C.S × Length

$= w \times \pi/4\, D^2 \times L$ ——— ①

Weight of hollow shaft, W_h = $w \times$ A of C.S × length

$= w \times \pi/4 [D_o^2 - D_i^2] \times L$

$= w \times \pi/4 [D_o^2 - (2/3 D_o)^2] \times L$

$= w \times \pi/4 [D_o^2 - 4/9 D_o^2] \times L$

$= w \times \pi/4 \times 5/9\, D_o^2 \times L$ ——— ②

Dividing eqn (1) by (2)

$$\frac{W_s}{W_h} = \frac{w \times \pi/4\, D^2 \times L}{w \times \pi/4 \times 5/9\, D_o^2 \times L} = \frac{9D^2}{5D_o^2}$$

$$= \frac{9}{5} \times \frac{(0.929 D_o)^2}{D_o^2}$$

$$\boxed{\frac{W_s}{W_h} = \frac{1.55}{1}}$$

Problem 5:

A solid cylindrical shaft is to transmit 300 KW power at 100 r.p.m

(a) If the shear stress is not to exceed $80 N/mm^2$. find its dia

(b) what percent of saving in weight would be obtained if the shaft is replaced by a hollow one whose internal diameter equals to 0.6 of the external diameter, the length, the material and max. shear stress being the same?

Given:

Power, P = 300 KW = 300×10^3 W

Speed, N = 100 r.p.m

max. shear stress, $\tau = 80 N/mm^2$

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Solution:

(a) D : dia. of Shaft.

$$P = \frac{2\pi N T}{60} \Rightarrow \frac{2\pi \times 100 \times T}{60} = 300 \times 10^3$$

$$\boxed{T = 2.86 \times 10^7 \text{ N-mm.}}$$

Torque, $T = \frac{\pi}{16} \tau D^3$

$$2.86 \times 10^7 = \frac{\pi}{16} \times 80 \times D^3$$

$$D = 121.8 \text{ mm}$$

$$\boxed{D \simeq 122.00 \text{ mm}}$$

(b) % Saving in weight.

let D_o = Ext. dia & D_i = int. dia. of hollow shaft.

$$D_i = 0.6 D_o$$

$$T = \frac{\pi}{16} \times \tau \times \frac{(D_o^4 - D_i^4)}{D_o}$$

Since torque transmitted by solid shaft and hollow shaft are the same

$$2.86 \times 10^7 = \frac{\pi}{16} \times 80 \times \frac{[D_o^4 - (0.6 D_o)^4]}{D_o}$$

$$1.82 \times 10^6 D_o = 0.8704 D_o^4$$

$$D_o^3 = 2090992.6$$

$$\boxed{D_o = 127.87 \text{ mm} \simeq 128.00 \text{ mm}}$$

$$D_i = 0.6 D_o$$

$$D_i = 0.6 \times 128$$

$$\boxed{D_i = 76.8 \simeq 77 \text{ mm}}$$

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Let W_s = weight of solid shaft.
W_h = weight of hollow shaft.

W_s = Weight density (w) × Volume
$= w \times \frac{\pi}{4} D^2 \times L$

Illy $w_h = w \times \frac{\pi}{4}(D_o^2 - D_i^2) \times L$

% Saving in weight $= \frac{W_s - W_h}{W_s} \times 100$

$$= \frac{w \times \pi/_4 D^2 \times L - w \times \pi/_4 [D_o^2 - D_i^2] \times L}{w \times \pi/_4 D^2 \times L} \times 100$$

$$= \frac{D^2 - (D_o^2 - D_i^2)}{D^2} \times 100$$

$$= \frac{122^2 - (128^2 - 77^2)}{122^2} \times 100$$

$\boxed{\text{% Saving in weight} = 29.55\ \%}$

Expression for torque in terms of Polar Moment of Inertia

Polar moment of Inertia of a plane area is defined as the ~~amount~~ moment of Inertia of the area about an axis ⊥r to the plane and passing through C.G of the area.

$$\boxed{J = \frac{\pi}{32} D^4}$$

Hence $\boxed{\frac{T}{J} = \frac{Z}{R} = \frac{C\theta}{L}}$

where,
C = modulus of rigidity.
θ = Angle of twist in radian.

Polar Modulus

+ It is defined as the ratio of the polar moment of inertia to the radius of the shaft.
+ Also called as torsional section modulus.

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$$Z_p = \frac{J}{R}$$

(a) Solid shaft, $J = \frac{\pi}{32} D^4$

$$Z_p = \frac{\frac{\pi}{32} D^4}{R} \Rightarrow \frac{\frac{\pi}{32} D^4}{D/2} = \frac{\pi}{16} D^3$$

(b) For hollow shaft, $J = \frac{\pi}{32} (D_o^4 - D_i^4)$

$$Z_p = \frac{\frac{\pi}{32} (D_o^4 - D_i^4)}{D_o/2} = \frac{\pi}{16 D_o} (D_o^4 - D_i^4)$$

Strength of a Shaft and torsional rigidity

* The max. torque (or) max. power the shaft can transmit is defined as the strength of shaft.

* Torsional rigidity (or) stiffness of the shaft is defined as the product of modulus of rigidity (C) and polar moment of inertia (J).

Torsional rigidity = $C \times J$

* It is also defined as the torque required to produced a twist of one radian per unit length of the shaft.

From $\frac{T}{J} = \frac{C\theta}{L}$

$$C \times J = \frac{T \times L}{\theta}$$

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Problem 6:

Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 r.p.m. Also determine the length of shaft if the twist must not exceed 1° over the entire length. The max. shear stress is limited to 60 N/mm². Take $C = 8\times10^4$ N/mm².

Given: Power, $P = 90$ kW $= 90\times10^3$ W

Speed, $N = 160$ r.p.m.

Angle of twist, $\theta = 1° =$

max. shear stress, $\tau = 60$ N/mm²

$C = 8\times10^4$ N/mm²

To find: Dia of solid shaft, $D = ?$

length of the shaft, $L = ?$

Solution: $P = \frac{2\pi N T}{60}$

$$T = \frac{P\times 60}{2\pi N} = \frac{90\times10^3\times60}{2\times\pi\times160}$$

$$T = 5371.4 \text{ N-m.}$$

$$\boxed{T = 537.4\times10^3 \text{ N-mm}}$$

$$T = \frac{\pi}{16}\times\tau\times D^3$$

$$53710.4\times10^3 = \frac{\pi}{16}\times60\times D^3$$

$$D^3 = \frac{53710.4\times10^3\times16}{\pi\times60}$$

$$D^3 = 455.938\times10^3$$

$$\boxed{D = 76.96 \text{ mm}}$$

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ii) Length of the Shaft

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

$$\frac{\tau}{R} = \frac{C\theta}{L} \Rightarrow \frac{60}{(76.96/2)} = \frac{8\times10^{4} \times \pi}{L\times180}$$

$$\boxed{\Rightarrow L = 895.47 \text{ mm}}$$

Problem 7:

A hollow shaft of dia 3/8 ratio (int. dia to outer dia) is to transmit 375 KW power at 100 r.p.m. The max. torque being 20% greater than the mean. The shear stress is not to exceed 60 N/mm² and twist in a length of 4 m not to exceed 2°. Calculate its ext & internal dia's which would satisfy both the above conditions. $C = 0.85\times10^{5}$ N/mm²

Given: $\frac{D_i}{D_o} = \frac{3}{8}$

$$D_i = \frac{3}{8}\times D_o$$

Power, $P = 375$ KW $= 375\times10^{3}$ W.

Speed, $N = 100$ r.p.m.

Max. Torque $T_{max} = 1.2\, T_{mean}$.

length, $L = 4$m $= 4000$ mm.

Max. twist, $\theta = 2^{\circ} \Rightarrow 2\times\frac{\pi}{180}$ rad.

Modulus of rigidity, $C = 0.85\times10^{5}$ N/mm²

To find: D_i & D_o

Solution: Power, $P = \frac{2\pi N T}{60}$ (Here $T = T_{mean}$).

$$T_m = \frac{P\times60}{2\pi N}$$

$$T_m = \frac{375\times10^{3}\times60}{2\times\pi\times100}$$

$$\boxed{T_m = 35810 \text{ N-m}}$$

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$T_{max} = 1.2\, T_m$

$T_{max} = 1.2 \times 35810$

$= 42972$ Nm

$\boxed{T_{max} = 42972 \times 10^3 \text{ Nmm}}$

(i) Diameter of the shaft when shear stress is not to exceed 60 N/mm².

$$T_{max} = \frac{\pi}{16} \times \tau \times \frac{(D_o^4 - D_i^4)}{D_o}$$

$$42972 \times 10^3 = \frac{\pi}{16} \times 60 \times \frac{[D_o^4 - (3/8 D_o)^4]}{D_o}$$

$\boxed{D_o = 154.97 \simeq 155 \text{ mm}}$

$D_i = 3/8\, D_o = 3/8 \times 155$

$\boxed{D_i = 58.1 \text{mm.}}$

(ii) Dia. of shaft when twist is not to exceed 2°

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\frac{42972 \times 10^3}{\pi/32\,(D_o^4 - D_i^4)} = \frac{(0.85 \times 10^5) \times 0.0349}{4000}$$

$\boxed{D_o = 156.65\text{mm} \simeq 157 \text{ mm}}$

$D_i = 3/8 \times 156.65$

$\boxed{D_i = 59 \text{ mm}}$

Problem 8:

A shaft ABC of 500 mm length and 40 mm ext. dia is bored, for part of its length AB to a 20 mm dia and for the remaining length to a 30 mm dia. If the shear stress is not ~~of~~ to exceed 80 N/mm², find the max. power, the shaft can transmit at a speed of 200 r.p.m. If the angle of twist in the length of 20 mm dia bore is equal to that in 30 mm dia bore, find the length of the shaft that has been bored to 20 mm & 30mm dia.

Total length, L = 500 mm.
Ext Dia, D_o = 40 mm.

τ = 80 N/mm^2

N = 200 r.pm.

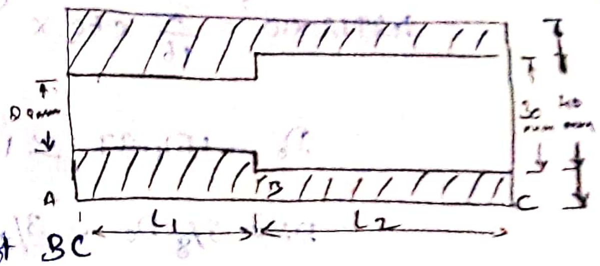
let length of shaft AB = L_1, Internal dia = D_{i1}, T_1 = torque

" " " BC = L_2, " " dia = D_{i2}, T_2 = torque

Solution: Torque transmitted by shaft AB

$$T_1 = \frac{\pi}{16}\tau\left[\frac{D_o^4 - D_{i1}^4}{D_o}\right]$$

$$T_1 = \frac{\pi}{16}\times 80\times\left[\frac{40^4 - 20^4}{40}\right]$$



$$\boxed{T_1 = 942500 \text{ N-mm}}$$

Illy Torque transmitted by shaft BC

$$T_2 = \frac{\pi}{16}\tau\left[\frac{D_o^4 - D_{i2}^4}{D_o}\right]$$

$$T_2 = \frac{\pi}{16}\times 80\times\left[\frac{40^4 - 30^4}{40}\right]$$

$$\boxed{T_2 = 687200 \text{ N-mm}}$$

!! Safe torque transmitted is the minimum torque of the two.

$\therefore$ Power transmitted (P) by T_2

$$P = \frac{2\pi NT}{60} = \frac{2\pi\times 200\times 687200}{60}$$

$$\boxed{P = 14390 \text{ W}}$$

From $\frac{T}{J} = \frac{C\theta}{L} \Rightarrow \theta = \frac{T\times L}{J.C}$

Safe torque and shear modulus are same for both safts.

$\therefore$ ~~AB~~ $\theta_1 = \frac{T}{C}\times\frac{L_1}{J_1}$ & $\theta_2 = \frac{T}{C}\times\frac{L_2}{J_2}$

$$\theta_1 = \theta_2$$

$$\therefore \frac{L_1}{J_1} = \frac{L_2}{J_2} \Rightarrow \frac{L_1}{\frac{\pi}{32}(40^4-20^4)} = \frac{L_2}{\pi/32(40^4-30^4)}$$

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$$\frac{L_1}{L_2} = \frac{(40^4 - 20^4)}{(40^4 - 30^4)} = 1.37$$

$L_1 = 1.37 L_2$

$L_1 = 1.37(500 - L_1)$

$L_1 = 1.37 \times 500 - 1.37 L_1$

$L_1 + 1.37 L_1 = 1.37 \times 500$

$\boxed{L_1 = 289 mm}$

$\boxed{L_2 = 211 mm.}$

Problem 9:

A Steel Shaft ABCD having a total length of 2.4m consists of three lengths having different sections as follows:

AB is hollow having outside & inner dias 80mm & 50mm

BC & CD are solid, BC of dia 80 mm & CD of dia 70mm.

If angle of twist is same for each section, determine the lengths and total angle of twist & max. shear stress in hollow portion is 50 N/mm². $C = 8.2 \times 10^4$ N/mm².

Given: Total length $L = 2.4m = 2400 mm$.

Shaft AB O.D, $D_o = 80 mm$

I.D, $D_i = 50mm$

length, $L_1 = ?$

Shaft BC Dia, $D_2 = 80 mm$ | shaft CD, Dia $D_3 = 70 mm$

length, $L_2 = ?$ | length $L_3 = ?$

Angle of twist $\theta_1 = \theta_2 = \theta_3$

Total angle of twist = ? = θ & $\tau = 50 N/mm^2$

$C = 8.2 \times 10^4 N/mm^2$

Solution:

To Affect angle of twist is same for all sections

$$\theta_1 = \theta_2 = \theta_3$$

$$\frac{T \times L_1}{J_1 \times C} = \frac{T \times L_2}{J_2 \times C} = \frac{T \times L_3}{J_3 \times C}$$

Since T & C are same for all sections

$$\frac{L_1}{J_1} = \frac{L_2}{J_2} = \frac{L_3}{J_3}$$

$$\frac{L_1}{\frac{\pi}{32}(D_o^4 - D_i^4)} = \frac{L_2}{\frac{\pi}{32} D_2^4} = \frac{L_3}{\pi/32\, D_3^4}$$

$$\frac{L_1}{\pi/32\,(80^4 - 50^4)} = \frac{L_2}{\pi/32 \cdot 80^4} = \frac{L_3}{\pi/32 \cdot 70^4}$$

$$\frac{L_1}{340 \times 10^6} = \frac{L_2}{4.02 \times 10^6} = \frac{L_3}{2.35 \times 10^6}$$

$L_1 = 1.44 L_3$

$L_2 = 1.71 L_3$

$L = L_1 + L_2 + L_3$

$2400 = 1.44 L_3 + 1.71 L_3 + L_3$

$2400 = 4.15 L_3$

$L_3 = 578.22$ mm

$L_1 = 832.64$ mm

$L_2 = 988.75$ mm.

For shaft AB, $\frac{\tau_1}{R} = \frac{C\theta_1}{L_1}$

$$\theta_1 = \frac{\tau_1 \times L_1}{(D_1/2) \times C} = \frac{50 \times 832.64}{(80/2) \times 8.2 \times 10^4}$$

$\theta_1 = 0.012$ rad. $= 0.727^\circ$

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Total angle of twist $\theta = \theta_1 + \theta_2 + \theta_3$

$\theta = 2 + 0.1$

$\boxed{\theta = 2.1^\circ}$

Springs

- The capacity to recover quickly from difficulties; toughness.
- The ability of substance or object to spring back into shape; elasticity.

* Springs are elastic bodies which absorb energy due to resilience.

* The absorbed energy released as an when required.

* The spring which absorbs greatest amount of energy for given stress, without getting permanently distorted is the best spring.

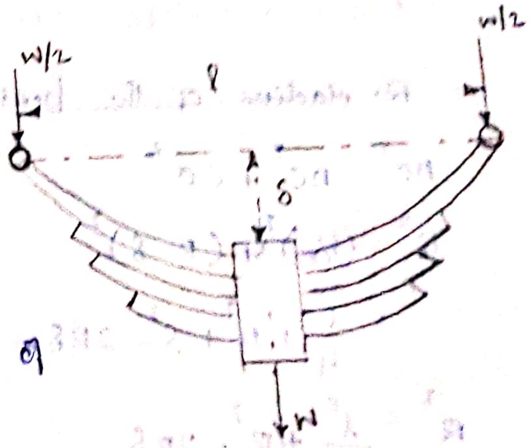
Types:

(i) Laminated (or) Carriage (or) leaf spring.

(ii) Helical Spring.

Carriage Springs.

* Used to absorb shocks, in railway wagons, coaches & road vehicles.



let b = width of each plate.

n = no. of plates.

l = Span of spring.

σ = max. bending stress

t = thickness of each plate.

W = point load acting at the centre of spring

δ = original deflection of the top spring.

Expression for max. bending stress developed.

* Load W acting on centre lower plate is shared equally by top pl

B.M at Centre = load at one end $\times l/2$

$$BM = W/2 \times l/2 = \frac{W.l}{4}$$

Moment of inertia, $I = \frac{bt^3}{12}$

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Relation of Bending stress (σ), bending moment (M) & moment of Inertia is given by.

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma}{y} \times I \Rightarrow \frac{\sigma \times \frac{bt^3}{12}}{t/2} \Rightarrow \frac{\sigma . bt^2}{6} \quad [\text{Here } y = t/2]$$

Total resisting moment by n plates $= n \times M = \frac{n.\sigma.bt^2}{6}$.

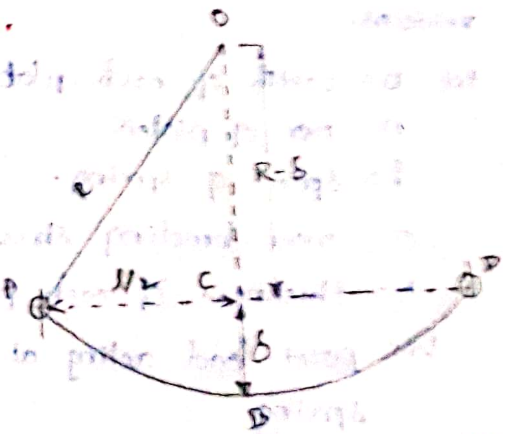
Since, total resisting moment = max. Bending moment.

$$\frac{W.l}{4} = \frac{n.\sigma.bt^2}{6}$$

$$\boxed{\sigma = \frac{6.W.l}{4.n.b.t^2} = \frac{3Wl}{2nbt^2}}$$

Expression for central deflection.

R = radius of the bent plates



$$AO^2 = AC^2 + CO^2$$

$$R^2 = (l/2)^2 + (R-\delta)^2$$

$$= \frac{l^2}{4} + R^2 + \delta^2 - 2R\delta$$

$$R^2 = \frac{l^2}{4} + R^2 - 2R\delta$$ (Neglect δ^2 as it is a small quantity)

$$2R\delta = \frac{l^2}{4}$$

$$\delta = \frac{l^2}{4 \times 2R} = \frac{l^2}{8R}$$

From relation b/w bending stress, modulus of elasticity and radius of curvature (R)

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$R = \frac{E \times y}{\sigma} \Rightarrow \frac{E \times t}{2\sigma}$$

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Substitute value of R in above eqn.

$$\delta = \frac{l^2 \times 2 \times \sigma}{8 \times E \times t}$$

$$\boxed{\delta = \frac{\sigma . l^2}{4E.t}}$$

Problem 1:

A leaf spring carries a central load of 3000N. The leaf spring is to be made of 10 steel plates 5 cm wide and 6mm thick. If the bending stress is limited to 150 N/mm². $E = 2 \times 10^5$ N/mm²

Find: (a) length of the spring.

(b) Deflection at the centre.

Given:

Central load (w) = 3000 N

No. of plates (n) = 10

width of plate (b) = 5cm = 50mm

thickness (t) = 6 mm

Bending stress (σ) = 150 N/mm²

Youngs modulus (E) = 2×10^5 N/mm²

To find:

length (l) = ?

Deflection (δ) = ?

Solution:

Bending stress $(\sigma) = \frac{3Wl}{2nbt^2}$

$$l = \frac{\sigma . 2n.bt^2}{3W} = \frac{150 \times 2 \times 10 \times 50 \times 6^2}{3 \times 3000}$$

$$\boxed{l = 600 \text{ mm}}$$

$$\text{Deflection } (\delta) = \frac{\sigma . l^2}{4.E.t} = \frac{150 \times 600^2}{4 \times 2 \times 10^5 \times 6}$$

$$\boxed{\delta = 11.25 \text{mm}}$$

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Helical Springs.

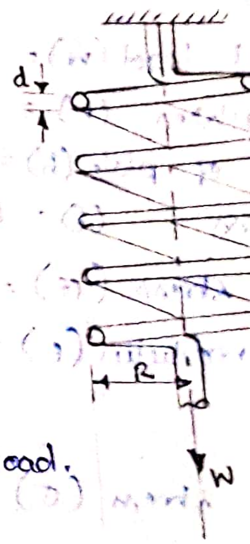
Thick wires coiled into a fet helix are called helical springs.

1. Closed coil
2. Open coil.

Close - Coiled Helical Springs.

* In such coils the pitch b/w two adjacent coils is small. (or)
* Helix angle is very small.
* Bending effect on spring is ignored as the helix angle is small.
* Hence only pure torsional stresses induced.

Exp. for max. shear stress induced in wire.



d = Dia. of spring wire.
p = Pitch of helical spring.
n = no. of Coils.
R = Mean radius of spring coil.
W = Axial load on spring.
C = Modulus of rigidity.
τ = max. shear stress induced in wire.
θ = Angle of twist in spring wire.
δ = Deflection of spring due to axial load.
l = length of wire.

Twisting moment on wire,

$$\boxed{T = W \times R} \quad \text{——— (i)}$$

Twisting moment actually given by.

$$\boxed{T = \frac{\pi}{16} \tau d^3} \quad \text{——— (ii)}$$

Equating (i) & (ii)

$$W \times R = \frac{\pi}{16} \tau d^3 \quad \text{(or)} \quad \tau = \frac{16 W \times R}{\pi d^3}$$

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Exp. for deflection of spring.

Now length of one coil = $2\pi R$

∴ Total length of the wire = Length of one coil × No. of coils.

$$l = 2\pi R \times n.$$

Since every section of wire is subjected to torsion,

Strain energy stored by spring $\boxed{U = \frac{\tau^2}{4C} \cdot \text{volume}}$

$$U = \left(\frac{16 W.R}{\pi d^3}\right)^2 \times \frac{1}{4C} \times (\text{Area of wire} \times \text{length})$$

$$U = \left(\frac{16 W.R}{\pi d^3}\right)^2 \times \frac{1}{4C} \times \left(\frac{\pi}{4} d^2 \times 2\pi R.n\right)$$

$$U = \frac{\cancel{256}^{16} W^2 R^2}{\cancel{\pi^2} d^{\cancel{6}4}} \times \frac{1}{\cancel{4}C} \times \frac{\cancel{\pi}}{\cancel{4}} d^2 \times 2\cancel{\pi} R.n$$

$$\boxed{U = \frac{32 W^2 R^3.n}{Cd^4}}$$

Work done on the spring = Avg. load × Deflection.

$$= \tfrac{1}{2}.W \times \delta$$

Equating the work done on spring to energy stored

$$\frac{1}{2} W.\delta = \frac{32 W^2 R^3.n}{Cd^4}$$

$$\boxed{\delta = \frac{64 W R^3 n}{Cd^4}}$$

Exp. for stiffness of spring.

S = load per unit deflection.

$$S = \frac{W}{\delta} = \frac{W}{\frac{64 W R^3 n}{Cd^4}} = \boxed{\frac{Cd^4}{64 R^3.n.}}$$

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Solid length is length of spring when there is no gap b/w coils.

Solid length = No. of coils × dia of wire
= n×d.

Problem 1:

A closely coiled helical spring is to carry a load of 500N. Its mean coil diameter is to be 10 times that of wire dia. Calculate the dia's. if max. shear stress in the material of spring is to be $80 N/mm^2$.

Given: $W = 500 N$ — To find: $D = ?$, $d = ?$

$D = 10.d$

$\tau = 80 N/mm^2$

Solution: $\tau = \frac{16 W R}{\pi d^3} \Rightarrow 80 = \frac{16 \times 500 \times (D/2)}{\pi . d^3}$

$$80 = \frac{16 \times 500 \times \left(\frac{10.d}{2}\right)}{\pi d^3}$$

$$80 = \frac{2500 \times 16 \times d}{\pi d^3}$$

$$d^2 = 159.15$$

$$\boxed{d = 12.6 mm}$$

$$D = 10 . d = 10 \times 12.6$$

$$\boxed{D = 126.15 mm}$$

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(1)

(11)

Problem 9:

A closely coiled helical spring of mean dia 20 cm is made of 3 cm dia rod and has 16 turns. A weight of 3 kN is dropped on this spring. Find the height by which the weight should be dropped before striking the spring so that the spring may be compressed by 18 cm. Take $C = 8\times10^4$ N/mm².

Given:

Mean dia, $D = 20$ cm
$D = 200$ mm
$R = 100$ mm

Wire dia, $d = 3$ cm
$= 30$ mm

No. of turns, $n = 16$

deflection, $\delta = 18$ cm
$\delta = 180$ mm.

$C = 8\times10^4$ N/mm².

Weight, $W = 3$ kN
$= 3\times10^3$ N

To find:

h = Height through which the weight W is dropped.

Solution: W = Gradual load applied which produces compression in spring equal to 180 mm.

$$\delta = \frac{64 W . R^3 . n}{C d^4}$$

$$180 = \frac{64 \times W \times 100^3 \times 16}{8\times10^4 \times 30^4}$$

$W = 11390.625$ N

Work done by the falling weight on spring = weight falling × $(h+\delta)$

$\boxed{= 3\times10^3 (h+180)}$ — (1)

Work done on spring
~~Energy stored~~ $= \frac{1}{2} W \times \delta$

$= \frac{1}{2}(11390.625)\times180$

$\boxed{= 1025156.25 \text{ N-mm.}}$ — (2)

Equating (1) & (2)

$3\times10^3(h+180) = 1025156.25$

$h+180 = 341.71$ mm

$\boxed{h = 161.7 \text{ mm.}}$

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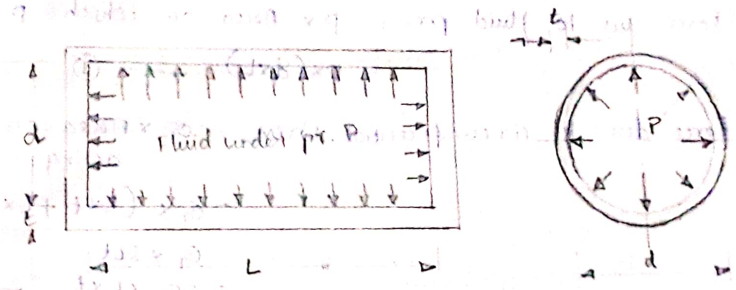
UNIT - V

THIN CYLINDERS, SPHERES AND THICK CYLINDERS

* If the thickness of the wall of the cylinder is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylinder vessel is known as thin cylinder.

* The stress distribution is assumed uniform over the thickness of the wall in case of thin cylinders.

Thin cylindrical vessel subjected to internal pressure.



where d = Internal dia of thin cylinder.
t = Thickness of the wall of cylinder.
P = Internal pr. of the fluid.
L = Length of the cylinder.

Stresses in a thin cylindrical vessel subjected to internal pressure.

The stresses that is set up in the thin cylindrical vessel due to the internal pr. of fluid are tensile.

1. Circumferential stress (or hoop stress) and
2. Longitudinal stress.

Expression for circumferential stress (or Hoop stress)

Exp. for hoop (or) circumferential stress σ_1

p = Internal pr. of fluid.
d = Internal dia of cylinder.
t = thickness of the wall.

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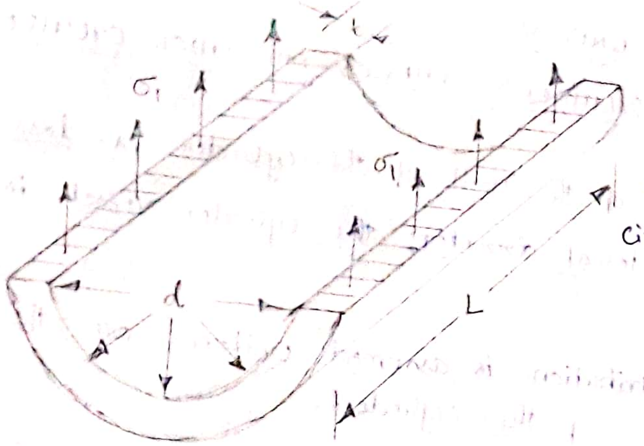


* Bursting happens when fluid pr. is more than resisting force due to circumferential stress

, when two forces are equal.

Force due to fluid pr. = P × Area on which P is acting.

= P×(d×L) —— ①

Force due to circumferential stress = σ_1 × Area on which σ_1 is acting

$= \sigma_1 \times (L\times t + L\times t)$

$= \sigma_1 \times 2Lt$

$= 2\sigma_1 \times L \times t$ —— ②

ide

Equating ① & ②, we get: $p\times d\times L = 2\sigma_1 \times L\times t$

$$\boxed{\sigma_1 = \frac{pd}{2t}} \quad \text{—— ③}$$

Exp. for Longitudinal stress. (σ_2).

* Bursting happens if the force due to fluid pr. acting on the ends of the cylinder. is more than resisting force due to longitudinal stress.

* Force due to fluid pr. = p× Area on which p is acting.

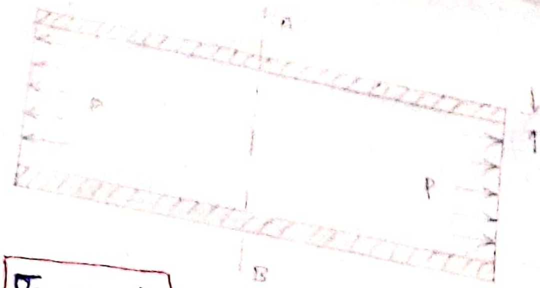
$= p \times \pi/4 \, d^2$

* ~~Force due to fluid pr.~~ Resisting force = σ_2 × Area on which σ_2 is acting.

$= \sigma_2 \times \pi d \times t$

Force due to fluid pr. = Resisting force.

$p\times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$.



$$\boxed{\sigma_2 = \frac{Pd}{4t}} \quad \text{---} \quad (ii)$$

The stress σ_2 is also tensile

$\sigma_2 = \frac{1}{2}\sigma_1$

Longitudinal stress = Half of circumferential stress.

* ∴ In the material of cylinder the permissible stress should be less than the circumferential stress.

Max. Shear stress.

* At any pt. in the material of the cylindrical shell, there are two principal stresses, namely a circumferential stress of magnitude (σ_1) acting ~~as to~~ circumferentially and a longitudinal stress of magnitude (σ_2) acting parallel to the axis of the shell.

* σ_1 & σ_2 are tensile and ⊥r to each other.

* $$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} \Rightarrow \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2}$$

$$= \frac{2Pd - Pd}{4t} \times \frac{1}{2}$$

$$\tau_{max} = \frac{Pd}{8t}.$$

Problem 1:

A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm^2. Determine
(i) Longitudinal stress in pipe &
(ii) Circumferential stress developed in pipe.

Sol: Given:

Inner dia, $d = 1.5$ m.
thickness, $t = 1.5$ cm $= 1.5 \times 10^{-2}$ m.
Internal fluid pr. $P = 1.2$ N/mm^2

Problem 2:

A thin cylinder of I.D 1.25 m contains a fluid at an internal pr. of 2 N/mm^2. Determine the max. thickness of cylinder if
(i) longitudinal stress is not to exceed 30 N/mm^2.
(ii) Circumferential stress not to exceed 45 N/mm^2.

Given:
I.D of Cylinder, $d = 1.25$ m
Internal pr., $P = 2$ N/mm^2
longitudinal stress $\sigma_2 = 30$ N/mm^2
Circumferential stress, $\sigma_1 = 45$ N/mm^2

To find:
Max. thickness of Cylinder, t

Solution:

$$\sigma_1 = \frac{Pd}{2t}$$

$$t = \frac{Pd}{2\sigma_1} = \frac{2 \times 1.25}{2 \times 45} = 0.0277\text{m}.$$

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(3)

$\sigma_2 = \frac{pd}{4t} \Rightarrow t = \frac{pd}{4\sigma_2}$

$t = \frac{2\times1.25}{4\times30} = \cancel{0.0280\,m}$ 0.0208

* Longitudinal (or) circumferential stresses induced in the material are inversely proportion to the thickness (t).
* Hence stress induced will be less if the value of 't' is more
* $\therefore \boxed{t = 0.0208\,m}$

Note:

i) If thickness of the cylinder is to be determined then eqn $\sigma_1 = \frac{pd}{2t}$ should be used.

(ii) If max. permissible stress is given, it should be taken circumferential stress (σ_1).

(iii) while using eqns (3) & (4), the units p, σ_1 and σ_2 should be same. They should be expressed either in N/mm^2 (or) N/m^2. Also the units of d and t should be same i.e. in m (or) mm.

Efficiency of Joint

* In riveted shell the circumferential & longitudinal stress are greater than in eqn (3) & (4).
* If efficiency is given.

η_l = Efficiency of longitudinal joint.

η_c = Efficiency of circumferential joint.

$$\boxed{\sigma_1 = \frac{pd}{2t\times\eta_l}} \quad (5) \qquad \text{and} \qquad \boxed{\sigma_2 = \frac{pd}{4t\times\eta_c}} \quad (6)$$

Note:

* Efficiency of a joint means efficiency of a longitudinal joint.
* If efficiency given, thickness is determined from eqn (5).

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Problem 8:

A boiler is subjected to an internal steam pr. of 2 N/mm². The thickness of boiler plate is 2.0 cm" and permissible tensile stress is 120 N/mm². Find out the max. dia when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%.

Given:

Internal steam pr. $p = 2 N/mm^2$

Thickness $t = 2.0$ cm

Permissible stress $= 120 N/mm^2$

Efficiency of longitudinal joint $\eta_l = 90\% = 0.90$

" " circumferential " $\eta_c = 40\% = 0.40$.

Solution

max. dia for circumferential stress.

$$\sigma_1 = \frac{Pd}{2t \times \eta_l} \Rightarrow 120 = \frac{2 \times d}{2 \times 2 \times 0.90}$$

$$\boxed{d = 216.0 \text{ cm}}$$

[longitudinal & circumferential stress are directly proportional to dia].

Max. dia for longitudinal stress.

$$\sigma_2 = \frac{Pd}{4t \times \eta_c} \Rightarrow 120 = \frac{2 \times d}{4 \times 2 \times 0.40}$$

$$\boxed{d = 192 \text{cm}}$$

Ans is d = 192 cm

Bcz if d = 216.0 cm the stress induced will be greater than the permissible stress.

Effect of Internal pressure on the dimensions of a thin cylindrical Shell.

Let E = Modulus of Elasticity
μ = poisson's ratio
δd = change in diameter due to stress.
δL = change in length.
δV = change in volume.
e_1 = Circumferential strain
e_2 = Longitudinal strain.

Then circumferential strain

$$e_1 = \frac{\sigma_1}{E} - \frac{\mu\sigma_2}{E}$$

$$= \frac{Pd}{2tE} - \frac{\mu Pd}{4tE}$$

$$\boxed{e_1 = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]} \quad \text{—— (1)}$$

Then longitudinal strain

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_1}{E}$$

$$= \frac{Pd}{4tE} - \frac{\mu Pd}{2tE}$$

$$\boxed{e_2 = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]} \quad \text{—— (2)}$$

Circumferential strain also given as.

$$e_1 = \frac{\text{Change in circumference}}{\text{Original Circumference}}$$

$$= \frac{\text{Final Circumference} - \text{original circumference}}{\text{Original circumference}}$$

$$e_1 = \frac{\pi(d + \delta d) - \pi d}{\pi d} = \frac{\pi d + \pi\delta d - \pi d}{\pi d}$$

Neglecting small quantities. $(\delta d^2)L$, $\delta L(\delta d^2)$ and $2d\,\delta d\,\delta L$, we get

$$e_1 = \frac{\pi\,\delta d}{\pi d}$$

$$\boxed{e_1 = \frac{\delta d}{d}} \quad \left[\frac{\text{Change in dia}}{\text{original dia}}\right] \quad \text{---} (3)$$

Equating the values of e_1

$$\frac{\delta d}{d} = \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right]$$

$$\boxed{\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]} = \text{change in dia} \quad \text{---} (4)$$

Illy longitudinal strain.

$$e_2 = \frac{\text{Change in length}}{\text{original length}} \Rightarrow \boxed{\frac{\delta L}{L}} \quad \text{---} (5)$$

Equating the values of e_2

$$\frac{\delta L}{L} = \frac{Pd}{2tE}\left[\frac{1}{2} - \mu\right]$$

$$\boxed{\delta L = \frac{PdL}{2tE}\left[\frac{1}{2} - \mu\right]} \quad \text{---} (6)$$

Volumetric Strain.

It is defined as the ratio of change in volume to original volume.

$$\text{Volumetric strain} = \frac{\delta V}{V}$$

Original Volume (V) = Area of cylindrical shell × Length.

$$= \frac{\pi}{4} d^2 \times L$$

Final Volume = Final area of cross section × Final length.

$$= \frac{\pi}{4}(d + \delta d)^2 \times (L + \delta L)$$

$$= \frac{\pi}{4}\left[d^2 + \delta d^2 + 2d\,\delta d\right] \times (L + \delta L)$$

$$= \frac{\pi}{4}\left[d^2 L + (\delta d^2)L + 2dL\,\delta d + \delta L d^2 + \delta L(\delta d^2) + 2d\,\delta d\,\delta L\right]$$

Neglecting small quantities. $(\delta d^2)L$, $\delta L(\delta d^2)$ and $2d\,\delta d\,\delta L$, we get.

Final Volume $= \frac{\pi}{4}[d^2L + 2d.L.\delta d + \delta L d^2]$

Change in Volume $(\delta V) = \frac{\pi}{4}[d^2L + 2.d.L.\delta d + \delta L d^2] - \frac{\pi}{4} d^2 \times L$

$$= \frac{\pi}{4}[2d.L.\delta d + \delta L d^2]$$

Volumetric strain $= \frac{\delta V}{V} = \frac{\frac{\pi}{4}[2.d.L.\delta d + \delta L d^2]}{\pi/4\, d^2 \times L}$

$$= \frac{2\delta d}{d} + \frac{\delta L}{L}$$

$$= 2e_1 + e_2$$

$$= 2 \times \frac{Pd}{2tE}\left[1 - \frac{\mu}{2}\right] + \frac{Pd}{2ET}\left[\frac{1}{2} - \mu\right]$$

$$= \frac{Pd}{2ET}\left[2 - \frac{2\mu}{2} + \frac{1}{2} - \mu\right]$$

$$= \frac{Pd}{2tE}\left[2 + \frac{1}{2} - \mu - \mu\right]$$

$$\boxed{\frac{\delta V}{V} = \frac{Pd}{2tE}\left[\frac{5}{2} - 2\mu\right]} \quad \text{——— (7)}$$

Also $\boxed{\delta V = V(2e_1 + e_2)}$ ——— (8)

Problem 1:

Calculate (i) the change in dia, (ii) change in length and (iii) change in volume, of a thin cylindrical shell 100 cm dia, 1 cm thick and 5 m long when subjected to internal pr. of $3 N/mm^2$. Take value of $E = 2 \times 10^5\ N/mm^2$ and poissons ratio $\mu = 0.3$.

Given:

dia, $d = 100$ cm.

Thickness, $t = 1$ cm

Length, $L = 5$ m

Internal pr. $p = 3 N/mm^2$

Youngs modulus, $E = 2 \times 10^5\ N/mm^2$

poissons ratio, $\mu = 0.3$

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Solution:

(i) Change in dia l, $\delta d = \frac{Pd^2}{2tE}\left[1 - \frac{\mu}{2}\right]$

$$\delta d = \frac{3 \times 100^2}{2 \times 1 \times 2 \times 10^5}\left[1 - \frac{0.30}{2}\right]$$

$$= \frac{3}{40}\left[1 - 0.15\right]$$

$$\boxed{\delta d = 0.06375 \text{ cm.}}$$

(ii) Change in length $(\delta L) = \frac{P.d.L}{2tE}\left[\frac{1}{2} - \mu\right]$

$$= \frac{3 \times 100 \times 500}{2 \times 1 \times 2 \times 10^5}\left[\frac{1}{2} - 0.30\right]$$

$$= \frac{3}{8} \times 0.20$$

$$\boxed{\delta L = 0.075 \text{ cm.}}$$

(iii) Change in volume $(\delta V) = V\left[2e_1 + e_2\right]$

$$\delta V = V\left[2.\frac{\delta d}{d} + \frac{\delta L}{L}\right]$$

$$\delta V = V\left[2.\frac{0.06375}{100} + \frac{0.075}{500}\right]$$

$$= V\left[0.001275 + 0.00015\right]$$

$$\delta V = 0.001425V$$

$$V = \frac{\pi}{4} \times 100^2 \times 500$$

$$V = 3926990.817 \text{ cm}^3$$

$$\delta V = 0.001425 \times 3926990.817$$

$$\boxed{\delta V = 5595.96 \text{ cm}^3}$$

Thin Spherical Shells

Internal dia = d.

thickness = t

Internal fluid pr. = P

[Fluid inside the shell has a tendency to split the shell into two hemispheres along x-x axis].

Force (P) has tendency to split. $= P \times \frac{\pi}{4} d^2$

The area resisting this force $= \pi . d . t$

∴ Hoop (or) circumferential stress $(\sigma_1) = \frac{\text{Force } P}{\text{Area resisting the force } P}$

$= \frac{P \times \frac{\pi}{4} d^2}{\pi . d . t}$

$= \frac{p.d}{4t}$

[The fluid also can split the sphere into two hemisphere along y-y axis].

$\sigma_2 = \frac{P.d}{4t}$. [The stress σ_2 will be at right angles to σ_1].

Problem 1:

A spherical vessel 1.5m dias is subjected to an internal pr. of $2 N/mm^2$. Find the thickness of the plate required if maximum stress is not to exceed $150 N/mm^2$. and joint efficiency is 75%.

Given:

Dia of shell, d = 1.5m, Fluid pr. $P = 2 N/mm^2$,

Stress in material, $\sigma_1 = 150 N/mm^2$, Joint Efficiency, $\eta = 75\% = 0.75$.

To find:

thickness of the plate, t = ?

Solution:

$\sigma_1 = \frac{p.d}{4t.\eta} \Rightarrow t = \frac{p.d}{4\sigma_1 . \eta} = \frac{2 \times 1.5 \times 10^3}{4 \times 0.75 \times 150}$

$\boxed{t = 6.67mm.}$

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